BROOKS' MENTAL ARITHMETIC.

KEY TO THE NORMAL MENTAL ARITHMETIC.

PHILADELPHIA:
SOWER, BARNES & CO.

METHODS

OF

TEACHING MENTAL ARITHMETIC,

AND

KEY

TO

THE NORMAL MENTAL ARITHMETIC.

CONTAINING ALSO MANY SUGGESTIONS AND METHODS FOR ARITHMETICAL CONTRACTIONS, AND A COLLECTION OF PROBLEMS OF AN INTERESTING AND AMUSING CHARACTER, FOR CLASS EXERCISE.

BY

EDWARD BROOKS, A. M.

PROFESSOR OF MATHEMATICS IN THE PENNSYLVANIA STATE NORMAL SCHOOL.

PHILADELPHIA:
SOWER, BARNES & CO.

MEARS & DUSENBERY, STEREOTYPERS.

PREFACE.

This little volume consists of four distinct parts:

First.—An exposition of some of the principles and methods of successful instruction in the science of Mental Arithmetic.

Second.—A fuller development of the principles presented in "The Normal Mental Arithmetic," by many remarks and suggestions, and by the solution of some of the more difficult problems.

Third.—The presentation of quite a number of methods of numerical computation by contractions, &c.; the object of which is to make pupils ready and accurate in the *mechanical* operations of Arithmetic.

Fourth.—A collection of a large number of problems of an amusing and interesting character, under the head of "Social Arithmetic," to be used to awaken interest in a class, or entertain a social circle.

With regard to the second part of the work, in which it may be considered a "Key," it will be noticed, that it differs from Keys, generally, in presenting many remarks and suggestions, and also in solving only a few of the more difficult problems, so that even if a class of pupils should happen to make use of it, in preparing their lessons, they will find great variety still among the unsolved problems, for the exercise of their own ingenuity.

The author hopes that the book may be found of value to the private student, to the teacher of Mental Arithmetic, and particularly to his numerous friends, who have shown their appreciation of his former works; and thus, in various ways, lend assistance to the great cause of the age,—the cause of Popular Education.

METHODS OF TEACHING

MENTAL ARITHMETIC.

CHAPTER I.

ARITHMETIC is the logic of numbers, and hence its truths and principles should be derived by logical processes. The four logical processes by which these truths and principles are obtained, are Analysis, Synthesis, Induction, and Deduction.

ANALYSIS.—*Analysis* is the process of resolving that which is complex into its elements. A watch is a complex being, consisting of wheels, springs, pins, hands, &c.. Now, if we take a watch and separate it into these parts, we are said to analyze it. A house is analyzed when we separate it into the brick, wood, stone, mortar, iron, &c., of which it is built. Analysis then means separating, taking to pieces, resolving the complex into the parts of which it is composed.

SYNTHESIS.—*Synthesis* is the reverse of Analysis. It is the process by which we form a complex object from simpler objects. Thus, if, after taking the watch apart by Analysis, we put the different parts together again, the process is called Synthesis. The building of a house is a synthetic process. The putting together of words to form a sentence, is also a process of Synthesis.

INDUCTION.—*Induction* is the logical process by which we derive general truths from particular ones. Thus, when we notice that heat expands iron, lead, copper, &c., we infer that heat will expand all metallic bodies, and this process of inferring the general truth is called Induction. As simple as this logical process may seem, it is of vast importance in nearly every department of science Its utility has been long recognised in the Natural and Philosophical sciences, but not in the science of Mathematics; it is, however, of very great importance even here, particularly in Arithmetic. In fact, it has long been used in Arithmetic, although apparently unconsciously, and also in a slight degree in Geometry and Algebra. Whenever a *general* truth or principle is derived in Arithmetic, without demonstration, it must be so derived by Induction. We have employed this process in Arithmetic as a logical method of procedure, and one which we claim to be of much importance. Its application and utility will be particularly seen in the treatment of Fractions.

DEDUCTION.—*Deduction* is the reverse of Induction. In Induction, as has been seen, we pass from the *special* to the *general;* in Deduction we reverse this process, and pass from the *general* to the *special.* Thus, having derived by Induction our general principle that heat expands all metallic bodies, we infer that heat will expand zinc and silver, since zinc and silver are metals. Induction is an ascending process—we go up from the parts to the whole; Deduction is a descending process—we descend from the whole to the particulars. Induction is a synthetic process, it puts together; while Deduction is an analytic process, it takes apart, or separates the few from the many or the whole.

INDUCTIVE TEACHING.—There is a method of teaching so intimately allied to the logical process of Induction, that educators have given it the name of the Inductive Method. It consists in leading the pupil along, step by step, to the conclusion, announcing such conclusion only after it is clearly seen by the pupil. Thus, in the definition of Addition, by the Inductive method, we would, by appropriate questions, lead the learner to a clear idea of Addition, and then, but not till then, we would give the name to the process, and also the definition of the name.

DEDUCTIVE TEACHING.—The Deductive method of instruction is the reverse of the Inductive method, and is so called because it resembles deductive logic. By this method a term, and the definition of it, are both given before the pupil has any idea of the thing defined, and then he is led to an understanding of it by appropriate examples and illustrations.

Without entering into a discussion of the merits of these two methods, the author would remark that he prefers the Inductive method for beginners, and the Deductive for more advanced pupils. In the oral and mental exercises of the "Primary Arithmetic" the Inductive method has been employed, and also, for the most part, in the "Normal Mental Arithmetic." In the more advanced departments of Mathematics, the Deductive method is preferred.

CHAPTER II.

WE have, in the previous chapter, given a brief outline of the general principles of Arithmetical Reasoning and Instruction. These principles will be found applied in the two little works of the author — THE NORMAL PRIMARY ARITHMETIC and THE NORMAL MENTAL ARITHMETIC.

Arithmetic, considered from the stand-point of teaching, should consist of Oral Arithmetic, Mental Arithmetic, and Written Arithmetic. It will be noticed that we use the terms *Mental* and *Written*, instead of the more frequently used ones, "Intellectual" and "Practical." Our reason for this is, that the terms are more appropriate, as a very little thought will show. The term "*Practical*," as applied to Arithmetic, is a misnomer—all Arithmetic should be practical. In one case we solve questions *mentally*, and in the other case we employ *written* characters to aid the mind in the computation; hence the propriety of the division of Arithmetic into *Mental* and *Written.*

Oral Arithmetic.—Oral Arithmetic consists of such instructions in the science and art of numbers as should precede the use of a text-book by the student. The learner needs such instructions for several reasons. First, pupils can learn Arithmetic before they can *read*, and hence, of course, before they can use a book. Secondly, even with pupils who can read, such exercises are a very valuable preparation to the study of the subject from the text-book.

The chief instrument to be employed in these oral exercises, is the "Arithmometer," or Numeral Frame, although *books*, *pens*, *pencils*, *grains of corn*, &c., form a valuable introduction to it.

In the "Normal Primary Arithmetic" we have given many suggestions for exercises of this kind, which we trust will be found valuable. The teacher can vary and modify them to suit the capacity and advancement of the class.

First Arithmetic.—The author believes that the Primary Arithmetic should be based upon the following principles, some of which have not previously been recognised in the preparation of such books.

1st. It should consist of both Mental and Written exercises, and not of either alone.

2d. Addition and Subtraction should be so presented in the Mental Exercises that they may be taught simultaneously, or that an additive process should be immediately reversed, thus giving rise to a subtractive one.

3d. In the Mental exercises Multiplication and Division should be taught simultaneously, Division being made to depend upon Multiplication, as it logically does, by reversing the multiplicative process.

4th. The pupils should derive their own "Multiplication Table," so that they may understand its meaning and use, being required to commit it to memory after they have obtained it.

5th. The *pictures* of objects, *marks*, *stars*, &c., should be omitted, since the objects themselves are better than their pictures or the marks, &c., and also, since the pupils should not leave the Oral Exercises until they can operate without the aid of objects or pictures.

6th. Principles and definitions should be taught *Inductively*, and the *methods* in fractions should be derived by inductive inferences from analytic processes.

Mental Arithmetic.—Although the first book, The Primary Arithmetic, should consist of a combination of Mental and Written exercises, yet it has been found most advantageous in teaching to separate them after the first book, having a complete course of Mental Arithmetic in one book, and a complete course of Written Arithmetic in another book.

The Mental Arithmetic, it is believed, should be based upon the following principles:

1st. It should be Analytical and Synthetical. Numbers are formed by Synthesis; hence, in the solution of problems, this synthetic process must often be reversed by the process of Analysis. Almost every solution, if properly given, involves the two processes of Analysis and Synthesis. Since the synthetic process is so easy after the analysis has been made, the two processes have been denoted by the term Analysis.

These two processes are included in the more general process of Comparison. Comparison is more properly the reasoning process, Analysis and Synthesis the mechanical processes.

The simplest process of Comparison which involves Analysis, is where we compare a number with one, a collection with the unit, since the relation is intuitively apprehended; and the simplest process of Comparison which involves Synthesis, is when we compare the unit with the collection. After the pupil becomes familiar with these elementary processes of Comparison, he begins to perceive relations existing between different collections of units—numbers—and he should be taught to apply those relations in the solutions of problems. This may be seen illustrated on pages 42 and 43 of the "Normal Mental Arithmetic."

2d. Fractions should be treated by Analysis and Synthesis and Induction, or more briefly by Comparison and Induction. In the treatment of fractions it is necessary to obtain methods for operating upon them, such as reducing to lowest terms, to common denominator, &c. These methods may be derived by Induction from the results obtained by the analytic process. This feature, which has not previously been introduced into Arithmetic, we deem a very valuable one, viewed either from the stand-point of science or teaching.

3d. The problems should be so varied that pupils will be forced to think for themselves. If all the problems under any class of problems, are like the one which is solved in the text-book, the pupils can take the solution given and apply it directly to them all, and thus no more thought is required than by the old method of working by Rule. To prevent this mechanical way of operating, we have taken much pains to give great variety to the problems.

The development of these features, and also of others which we deem of much value, will be seen by an examination of the Key, and also of the work itself.

OBJECT OF THE KEY.

This Key is designed to show the manner in which Mental Arithmetic may be successfully taught. In pursuing this object, many remarks and suggestions have been given, quite a number of solutions presented as models, and remarks given for the solution of some of the more difficult problems. The object has not been, as in Keys generally, to solve the difficult problems merely, and give results, but to suggest to teachers Methods of Teaching, which will be of use to them in the school-room.

When a solution is given as a model for the recitation-room, we head it with "SOL.;" but when suggestions merely are given, we use the heading "SUG." Remarks are indicated by the abbreviation "REM."

CHAPTER III.

METHODS OF RECITATION.

THE attention of teachers is respectfully solicited to the following Methods of Recitation. Some of them are preferable to others, but all may occasionally be used with advantage.

COMMON METHOD.—By this method the problems are read by the teacher and assigned promiscuously, the pupils not being permitted to use the book during recitation, nor retain the conditions of the problems by means of pencil and paper, as is sometimes done. The pupil selected by the teacher arises, repeats the problem, and gives the solution, at the close of which the mistakes that may have been made should be corrected by the class or teacher.

SILENT METHOD.—By this method the teacher reads a problem to the class, and then the pupils silently solve it, indicating the completion of the solution by the upraised hand. After the whole class, or nearly the whole class, have finished the solution, the teacher calls upon some member, who arises, repeats the problem, and gives the solution as in the former method.

By this method the whole class must be exercised upon every problem, thus securing more discipline than by the preceding method. It, however, requires more time than the first; hence, not so many problems can be solved at a recitation. We prefer the first method for advanced pupils, and the second, at least a portion of the time, with younger pupils.

CHANCE ASSIGNMENT.—This method differs from the first only in the assignment of the problems. The teacher marks the number of the lesson and the number of the problem, upon small pieces of paper, which the pupils may take out of a box passed around by the teacher or some member of the class. The teacher then, after reading a problem, instead of calling upon a pupil, merely gives the number of the problem, the person having the number arising, repeating, and solving it. By this method the teacher is relieved of all responsibility with reference to the hard and easy problems, and it is also believed that better attention is secured with it. It is particularly adapted to reviews and public examinations.

DOUBLE ASSIGNMENT.—By this method the pupil who receives the problem from the teacher, arises, repeats it, and then assigns it to some one else to solve. It may be combined with either the first or second methods. The objects of this method are variety and interest.

METHOD BY PARTS.—By this method different parts of the same problem are solved by different pupils. The teacher reads the problem and assigns it to a pupil, and after he has given a portion of the solution, another is called upon, who takes up the solution at the point where the first stops; the second is succeeded in like manner by a third; and so on, until the solution is completed. The object of this method is to secure the attention of the whole class, which it does very effectually. It is particularly suited to a large class consisting of young pupils.

UNNAMED METHOD.—By this method the teacher reads and assigns several problems to different members of the class, before requiring any solutions, after which those who have received problems are called upon in the order of assignment for their solutions. The advantages of this method are, first, the pupil, having some time to think of the problem, is enabled to give the solution with more promptness and accuracy, and secondly, the necessity of retaining the numbers and their relations in the mind for several minutes, affords a good discipline to the memory.

CHOOSING SIDES.—This is a modification of the old spelling class method, and is one calculated to elicit a very great degree of interest. By it two pupils, appointed by the teacher, select the others, thus forming two parties for a trial of skill, as in a game of cricket or base ball. The problems may be assigned alternately to the sides, by the teacher, by chance, by the leaders of the sides, or in any other way that may be agreed upon by the teacher and class.

In regard to these methods, the first, second, and third are probably the best for the usual recitations, but the other methods can very profitably be employed with younger classes, or, in fact, with any class, to relieve monotony and awaken interest. With advanced pupils we prefer the first method, or the first combined with the third.

ERRORS TO BE AVOIDED.

There is a large number of errors to which pupils in every section of the country are liable, a few of which we will mention. We classify them as errors of Position and errors of Expression.

ERRORS OF POSITION.—Pupils are exceedingly liable to assume improper positions and awkward attitudes during recitation, such as leaning on the desk or against the wall, putting the foot upon a seat, jamming the hands in the pockets, particularly when the problem is hard, playing with a button, watch chain, &c. All of these faults should be carefully guarded against, for reasons so obvious that they need not be mentioned. An erect and graceful carriage, aside from its relation to health, is of advantage to every lady and gentleman.

Errors of Expression.—Under this head we include errors of Articulation, Pronunciation, Grammar, &c. There is quite a large number of words which pupils in their haste mispronounce, and also quite a large number of combinations, which by a careless enunciation make ridiculous sense, or nonsense. We will call the attention to a few of them, suggesting to the teacher to correct these and others he may notice.

"*And*" is often called "*an*;" "*for*" is called "*fur*;" "*of*" is pronounced as if the *o* was omitted; words commencing with *wh*, as when, which, where, &c., are pronounced as if spelled "*wen*," "*wich*," "*were*," &c.

"*Gave him*" is called "*gavim*;" "*did he*" is called "*diddy*;" "*had he*" is called "*haddy*;" "*give him*" is called "*givim*;" "*give her*" is called "*giver*;" "*which is*" is often changed into "*witches*;" and "*how many*" is frequently transformed into "*hominy*." "How many did each earn" is often rendered "hominy did *e churn*."

A very common error, and one exceedingly difficult to correct, is involved in the following solution: "If 2 apples cost 6 cents, one apple will cost *the* ½ of 6 cents, which *are* 3 cents." Here "*the*" is superfluous, and "*are*" is ungrammatical.

The following is a frequent error: "*If one apple cost* 3 *cents, for* 12 *cents you can buy as many apples as* 3 *is contained in* 12, *which are* 4 *times*." The objections are, first, 3 is not contained any *apples* in 12; secondly, the result obtained is *times*, when it should be *apples*, or a *number* which applies to both times and apples. The solution should be, "*You can buy as many apples for* 12 *cents as* 3 *is contained times in* 12, *which are* 4."

With regard to *is* and *are*, it is not easy to determine which should be used in some cases in Arithmetic. I am rather inclined to think that it would be better to use the singular form always when the subject is either an abstract or a concrete number; thus, 8 is 2 times 4, or 8 apples is 2 times 4 apples. But, since custom sanctions the use of "*are*" with a concrete number as a subject, we have adhered to that form. There is some authority for using "*is*" in the "Multiplication Table," and I think it would be better if the singular form was universally adopted. We have adopted the following rule in determining the form of the verb when it has a *numerical* subject: If the idea is plural use "*are;*" if the idea is that of a whole—singular—use "*is.*"

Pupils have some difficulty in knowing how to read such expressions as $\$\frac{4}{3}$. They object to saying "$\frac{4}{3}$ *dollars,*" since there are not enough to make *dollars*, and they also object to saying "$\frac{4}{3}$ *of a dollar,*" since there are only 3 thirds in a dollar. The correct reading is undoubtedly the second, remembering that $\frac{4}{3}$ is an *improper* fraction.

The following error is almost universal: $2\frac{3}{4}$ apples is read "2 *and* 3 *fourth apples,*" instead of "2 *and* 3 *fourths apples.*" The expression "$\frac{3}{4}$ *times*" is sanctioned by custom, although it is not in accordance with grammatical principles. It is rather more convenient than the expression $\frac{3}{4}$ *of a time,* although evidently a violation of the rules of language.

But it is unnecessary for us to swell this list larger. A little care on the part of the teacher will detect a large number of errors, similar to those we have noticed, and we suggest that the attention given in this direction will be time profitably employed.

KEY.

SECTION I.

LESSON I.

A FEW of the more difficult problems in this section may be omitted the first time in going over by a class of young pupils. They are introduced for the purpose of interesting older pupils, that they may be induced to study "Simple Addition," in a Mental Arithmetic.

REMARK.—This lesson is so easy that it is not necessary to give the solutions of any of the problems. The 15th problem is designed as a puzzle, as will readily be seen.

LESSON II.

41. SOL.—Since, when he sold 10 none remained, before selling these 10 he had 10, but 6 of these he bought, hence before buying them he had 10 minus 6, or 4.

54. SOL.—If Hiram gave Oliver 10 cents, and Oliver gave Hiram 6 cents, Oliver had 10 — 6, or 4 cents more than at first, and Hiram had 4 less than at first; hence Oliver had 26 + 4 or 30 and Hiram had 26 — 4 or 22.

57. SOL.—If A sold 30 and then bought 12, he had 30 — 12, or 18 less than at first, and if by selling 30 more, none remained, he had at first 18 + 30, or 48.

SUGGESTION.—The 57th may also be solved as follows:

SOL.—In all he sold 30 + 30, or 60, but 12 of these he bought, hence he had at first 60 — 12, or 48.

58. SUG.—After A gave B 10 and B gave A 6, A had 4 less and B 4 more than at first; hence A had 26 and B 34. Then after B lost a certain number, 26 — 12, or 14, equals B's number; hence B lost 34 — 14, or 20.

LESSON III.

38. SUG.—Six times what they both have is 90; hence C has 90 — 10, or 80.

40. SUG.—After A gave B 10 and B gave A 20, A had 10 more and B had 10 less than at first; hence A had 50 and B had 30. Then after B lost a certain number he had twice 10, or 20; hence he lost 30 — 20, or 10.

LESSON IV.

NOTE.—The problems after the 36th may be assigned to the class and recited as the other problems, or to give variety to the exercise the pupils need not repeat the problem, but name the result as soon as the problem is announced by the teacher. It will be well for the teacher to give quite a number of such problems, as the exercise will be found very valuable.

LESSON V.

31. SOL.—Twice a number, plus 3 times the number equals 5 times the number, minus 4 times the number equals once the number, plus twice the number equals 3 times the number.

37 & 38. REM.—These problems and others like them may be made very interesting to the pupils by the teacher's stating to the class that he can tell the result which they obtain, no matter what number they begin with. This can always be done, and the reason is as follows: We require the pupils to multiply and divide until we reduce to the number they begin with, we then (not knowing, of course, what number they have) require them to add some number to the number that they have, and then require them to subtract the number they thought of, which leaves the number we told them to add, and we now know what number they have; we can, therefore, tell them to multiply or divide by anything we choose, and knowing the number they have, we can readily name the product or quotient. This little puzzle, even with an advanced class, I have known to elicit a very high degree of interest. When it is well understood, the teacher may show the class that they need not reduce it to once the number before adding the number which he gives them, provided they subtract as many times the number from the sum.

SECTION II.

NOTE.—This section treats of fractions by using the fractional *word*, but not the expression consisting of two figures with a line between them. This is regarded as a valuable preparation to the more general treatment of the following section.

LESSON I.

7. Sol.—If 1 yard of cloth cost 16 cents, 1 half of a yard of cloth will cost 1 half of 16 cents, which is 8 cents.

25. Sol.—If John had 21 cents and gave 2 thirds of them to Susan, Susan received 2 thirds of 21 cents; 1 third of 21 cents is 7 cents, and 2 thirds of 21 cents are 2 times 7 cents, which are 14 cents.

LESSONS II. & III.

No Suggestions or Solutions necessary.

LESSON IV.

19. Sug.—He sold 2 tenths + 3 tenths + 4 tenths, which are 9 tenths of his number, hence there remained 10 tenths — 9 tenths, which is 1 tenth of his number, that is, 1 tenth of 20 sheep, which is 2 sheep.

30. Sug.—A gave B 20 tenths of a dollar, then 20 tenths + 5 tenths, which are 25 tenths, equals 1 half of what I had, hence I then had 50 tenths of a dollar, and at first 56 tenths of a dollar.

38. Sug.—One yard cost 4 tenths of a dime, hence for 12 tenths of a dime 3 yards may be bought.

LESSON V.

Note.—The previous lessons in fractions were needed to prepare the student for the solution of the problems in this lesson. Some authors introduce problems similar to those in this lesson previous to any exercises in fractions, but such an arrangement is evidently illogical.

47. Sug.—The lemons were worth 24 cents, for which you could buy 6 oranges at 4 cents each; hence there remained 9 — 6, or 3 oranges.

LESSON VI.

Note.—The problems in this lesson are so simple that no solutions need be given in the Key. The author would remark that problems similar to the 29th and 30th should frequently be given to the class. They afford an excellent exercise for acquiring rapidity and accuracy of mental computation. The teacher can readily extemporize them, making them as complicated as the advancement of the class may allow.

LESSON VII.

Note.—The solution given in the Mental Arithmetic for the 24th is the reverse of that given for the first problem of this lesson, and since the 24th problem is the reverse of the first, the solution given seems most appropriate. The following solution to problems of this class is, however, preferred by some teachers.

24. Sol.—In *one* there are *three* thirds, hence one third of the number of thirds equals the number of ones; 1 third of 6 is 2; therefore, in 6 thirds there are 2 ones.

LESSON VIII.

NOTE.—The solution of the first problem is sometimes given thus: If 3 is one half of some number, two halves of that number is two times 3, or 6. The solution given in the Arithmetic is perhaps rather more logical.

30. SOL.—If $10 is 1 third of 6 times the cost of the dress, 3 thirds of 6 times the cost, or 6 times the cost of the dress is 3 times $10, which are $30; if 6 times the cost of the dress is $30, once the cost of the dress is 1 sixth of $30, which is $5.

36. SOL.—If 1 third of 1 half of some number is 8, 3 thirds of one half of that number is 3 times 8, or 24; if 1 half of some number is 24, 2 halves of the number equals 2 times 24, which are 48.

LESSONS X. & XI.

NOTE.—The method of solution in the previous part of this section has been to pass from a collection of things or parts to the single thing or part, and then pass from the single thing or part to another collection, thus passing from one collection to another. The process involves the logic of *comparison*, and is very easy, because the relation between a unit and a collection, or the relation between a collection and the unit, is so evident that it is readily apprehended. The pupils are now prepared to discover relations existing between different collections of units, that is, between different numbers, and they should be led to apply these relations in the solutions of problems. This is the object of Lessons X. and XI., and we commend them to the careful attention of the teacher. After the pupil is familiar with the method of running to the unit, it is a waste of time to require him to do so every time he wishes to compare two numbers.

LESSON XII.

NOTE.—This we deem a valuable lesson in preparing for fractions, although not usually introduced into works on Mental Arithmetic. The pupils should be drilled upon it until they are entirely familiar with it. The results will, of course, since the numbers are small, be determined by inspection. The pupils should be required to frame definitions for themselves from the suggestive statements made in the lesson.

SECTION III.

LESSON I.

1. SOL.—In one there are $\frac{2}{2}$ and in 3 there are 3 times $\frac{2}{2}$, which are $\frac{6}{2}$, and $\frac{6}{2}$ plus $\frac{1}{2}$ are $\frac{7}{2}$. Therefore, in $3\frac{1}{2}$ there are $\frac{7}{2}$.

NOTE.—One of the prominent features of this book is the derivation of *methods* in fractions from the results of analytic processes. Unless such methods are derived it would be necessary to give the analysis in full for every problem, which would become exceedingly tedious as the problems became complicated. Our plan then is, first to require the analytic solution, and after the pupils are familiar with such solution, to derive a rule, or *method* we prefer to call it, by which the results may be obtained more briefly.

After the pupils have analyzed a few of the problems in the first part of this lesson, lead them to see that they can reduce mixed numbers to fractions by *multiplying the entire part by the denominator of the fractional part, adding the numerator of the fractional part to the prod-*

uct, and using the result as the numerator of the fraction.

A similar remark applies to the problems from the 18th to the 26th, inclusive. The *method* is, *Divide the numerator by the denominator; if there is a remainder place it over the denominator, and unite this fraction to the quotient obtained by the division.*

LESSON II.

15. Sol.—It is evident that these fractions may be reduced to 12ths. *One* equals $\frac{12}{12}$, hence $\frac{1}{3}$ equals $\frac{1}{3}$ of $\frac{12}{12}$, which is $\frac{4}{12}$, and $\frac{2}{3}$ equals 2 times $\frac{4}{12}$, or $\frac{8}{12}$, &c.

Note.—Having derived a *method*, we may now omit the analysis, and proceed with this method as follows.

36. Sol.—To reduce $\frac{1}{2}$ to twentieths we multiply both numerator and denominator by 10, and have $\frac{1}{2}$, equal to $\frac{10}{20}$; to reduce $\frac{3}{4}$ to twentieths we multiply both numerator and denominator by 5, and have $\frac{3}{4}$, equal to $\frac{15}{20}$, &c.

46. Sol.—If 60 cents was $\frac{5}{4}$ of $\frac{1}{2}$ of what he then had, $\frac{1}{4}$ of $\frac{1}{2}$ of what he then had was $\frac{1}{5}$ of 60 cents, which is 12 cents, and $\frac{4}{4}$ of $\frac{1}{2}$ of what he then had was 4 times 12 cents, or 48 cents; if 48 cents was $\frac{1}{2}$ of what he then had, $\frac{2}{2}$ of what he then had equals 2 times 48 cents, which are 96 cents; hence before he found 60 cents he had 96 cents — 60 cents, or 36 cents. Therefore, &c.

48. Ans.—Henry had 45 cents, and his sister had 20 cents, at first.

LESSON III.

Note.—We analyze the problems to the eleventh, and then derive a *method*, which we apply in the solution of the problems which follow.

15. Sol.—To reduce $\frac{6}{10}$ to fifths we divide both numerator and denominator by 2, and we have $\frac{6}{10}$, equal to $\frac{3}{5}$, &c.

23. Sol.—To reduce $\frac{4}{8}$ to its lowest terms we divide both numerator and denominator by the greatest number that will divide them both without a remainder, which number is 4, and we have $\frac{4}{8}$, equal to $\frac{1}{2}$, &c.

37. Sol.—$4\frac{4}{5}$ melons $= \frac{24}{5}$ melons. If $\frac{24}{5}$ melons are worth 6 lemons, $\frac{1}{5}$ of a melon is worth $\frac{1}{24}$ of 6 lemons, which is $\frac{6}{24}$, or $\frac{1}{4}$ of a lemon, and $\frac{5}{5}$ of a melon is worth 5 times $\frac{1}{4}$, or $\frac{5}{4}$ of a lemon, and 7 melons are worth 7 times $\frac{5}{4}$, which are $\frac{35}{4}$, or $8\frac{3}{4}$ lemons.

LESSON IV.

Note.—This lesson treats of the addition and subtraction of fractions. We commence by adding and subtracting fractions having a common denominator, then by problems from 10 to 16 suggest the reduction to a common denominator, after which we reduce to common denominator by the *method* derived in Lesson II., and then add or subtract as the problem may require.

17. Sol.—$\frac{1}{2}$ is equal to $\frac{3}{6}$, and $\frac{1}{3}$ is equal to $\frac{2}{6}$; $\frac{3}{6}$ plus $\frac{2}{6}$ are $\frac{5}{6}$. Therefore the sum of $\frac{1}{2}$ and $\frac{1}{3}$ is $\frac{5}{6}$.

47. Sol.—If Thomas had $\frac{1}{3}$ of a dollar and found $\frac{2}{5}$ of a dollar, he then had $\frac{1}{3}$ of a dollar plus $\frac{2}{5}$ of a dollar; $\frac{1}{3} = \frac{5}{15}$ and $\frac{2}{5} = \frac{6}{15}$, $\frac{5}{15} + \frac{6}{15} = \frac{11}{15}$. Therefore, &c.

50. Sug.—$\frac{3}{8}$ of the sum plus $\frac{1}{2}$ of the sum equals $\frac{7}{8}$ of the sum, which was \$21; hence the sum was \$24.

78. Sol.—$\frac{5}{5}$ of the number diminished by $\frac{2}{5}$ of the number equals $\frac{3}{5}$ of the number, which equals 36; if $\frac{3}{5}$ of the number equals 36, &c.

Note.—The pupil may be led to see that when we wish to add two fractions having a unit for the numerator, we may take the *sum* of their denominators for the numerator of the sum, and the product of the denominator for the denominator of the sum; also, that the difference of two fractions, whose numerators are a unit, is the *difference* of their denominators divided by their product. Thus, $\frac{1}{4} + \frac{1}{5} = \frac{5+4}{4 \times 5} = \frac{9}{20}$, and $\frac{1}{4} - \frac{1}{7} = \frac{7-4}{7 \times 4} = \frac{3}{28}$.

LESSON V.

4. SOL.—If a boy lost 4 marbles and found 10, he then had 10 — 4, which is 6 more than at first, which equals $\frac{3}{2} - \frac{2}{2}$, or $\frac{1}{2}$ of what he had at first, &c.

14. SUG.—We find \$24 was what remained, which equals $\frac{5}{5}$ minus $\frac{2}{5}$, which is $\frac{3}{5}$ of what he had at first, &c.

24. SUG.—If he gave $\frac{1}{6}$ of $\frac{6}{7}$ away, there remained the difference between $\frac{6}{6}$ of $\frac{6}{7}$ and $\frac{1}{6}$ of $\frac{6}{7}$, which is $\frac{5}{6}$ of $\frac{6}{7}$.

28. SUG.—$\frac{5}{6}$ of $\frac{6}{7}$ equals $\frac{5}{7}$. If 60 feet is $\frac{5}{7}$ of the length of the shadow, diminished by 20 feet, 60 feet plus 20 feet, which are 80 feet, is $\frac{5}{7}$ of the length of the shadow, &c.

NOTE.—In solving problems like the first in this lesson, some say "let $\frac{5}{5}$ equal what he had at first." This is *absurd.* $\frac{5}{5}$ of what he has will equal it whether you *let* it or not.

LESSON VI.

35. ANS.—By dividing the denominator of $\frac{3}{4}$ by 2.

36. ANS.—By dividing the denominator of $\frac{5}{6}$ by 3.

39. SOL.—Dividing the denominator by 4 we have 4 times $\frac{5}{8}$ equals $\frac{5}{2}$, or $2\frac{1}{2}$.

59. SOL.—$2\frac{2}{3} = \frac{8}{3}$; 3 times $\frac{8}{3}$ miles equals 8 miles; if twice the distance is 8 miles, &c.

NOTE.—Hereafter, when a fraction is to be multiplied by a number which will divide the denominator, *insist* upon pupils dividing the denominator, instead of multiplying the numerator. This will require the continual attention of the teacher, for pupils adhere to the method of multiplying the numerator with wonderful tenacity.

LESSON VII.

42. SUG.—After selling $\frac{3}{4}$ of $\frac{8}{9}$ of a barrel, there remained $\frac{1}{4}$ of $\frac{8}{9}$ of a barrel, &c.

NOTE.—Pupils will naturally solve this by obtaining $\frac{3}{4}$ of $\frac{8}{9}$, and then subtracting this from $\frac{8}{9}$. The solution indicated is much more concise.

LESSON VIII.

NOTE.—The problems in this lesson as far as to the 13th should be solved like the 1st, then by means of the 13th and 14th we derive a *method*, which we apply to the solution of those which follow.

The first may also be solved thus: $\frac{1}{4}$ is equal to $\frac{3}{12}$, and $\frac{1}{3}$ of $\frac{3}{12}$ is $\frac{1}{12}$; hence $\frac{1}{3}$ of $\frac{1}{4}$ is $\frac{1}{12}$. The solution given in the Arithmetic is preferred, since it involves the *principle* of obtaining a part of a fraction, which the other solution does not.

15. SOL.—By multiplying the numerators together for the numerator of the result, and the denominators for the denominator, we find $\frac{1}{2}$ of $\frac{1}{3}$ to equal $\frac{1}{6}$.

21. SOL.—If a man, owning $\frac{1}{8}$ of a farm, sold $\frac{1}{6}$ of it to his neighbor, his neighbor received $\frac{1}{6}$ of $\frac{1}{8}$, which is $\frac{1}{48}$ of it.

NOTE.—The 30th to the 37th we solve like the 29th. At the 37th we derive the *method*, and apply this method to the solution of those which follow.

38. Sol.—By multiplying the numerators together for the numerator, and the denominators for the denominator of the result, we find $\frac{3}{8}$ of $\frac{3}{9}$ equals $\frac{9}{72}$, &c.

42. Sol.—If Johnson, having $\frac{2}{7}$ of a melon, gave $\frac{2}{3}$ of it to Martin, there remained $\frac{3}{3}$ of $\frac{2}{7}$ minus $\frac{2}{3}$ of $\frac{3}{7}$, which is $\frac{1}{3}$ of $\frac{3}{7}$, which equals $\frac{1}{7}$. Therefore, &c.

43. Sol.—If I had $\frac{3}{4}$ of a bushel of apples and gave $\frac{3}{4}$ of them away, there remained $\frac{4}{4}$ of them minus $\frac{3}{4}$ of them, which is $\frac{1}{4}$ of them; hence there remained $\frac{1}{4}$ of $\frac{3}{4}$, which is $\frac{3}{16}$ of a bushel.

50. Sug.—If she *shared* them *with* her schoolmates, they were divided equally between herself and 5 schoolmates, or 6 persons; hence each received $\frac{1}{6}$ of $\frac{2}{5} = \frac{1}{15}$ of a pound.

54. Sug.—It arose $\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{6}$, and then was $\frac{1}{2} + \frac{1}{6} = \frac{2}{3}$ of the first distance from the ground; it fell $\frac{1}{4}$ of $\frac{1}{6} = \frac{1}{24}$, and then was $\frac{2}{3} - \frac{1}{24} = \frac{5}{8}$ of whole distance from the ground.

LESSON IX.

12. Ans.—Multiply the dividend by the fraction obtained by inverting the terms of the divisor.

Note.—Assign a few of the problems which have been solved by the Analysis, and let the pupil apply the Method.

13. Sol.—If a yard of cloth cost $\frac{3}{5}$ of a dollar, for \$12 you can buy as many yards as $\frac{3}{5}$ is contained times in 12, which are $12 \times \frac{5}{3} = \frac{60}{3}$, or 20.

Note.—The 13th problem may also be solved by the following analytic process. Let the pupil understand both.

13. Sol.—If \$$\frac{3}{5}$ will buy one yard, \$$\frac{1}{5}$ will buy $\frac{1}{3}$ of a yard, and \$$\frac{5}{5}$ will buy 5 times $\frac{1}{3}$, which are $\frac{5}{3}$ of a yard, and \$12 will buy 12 times $\frac{5}{3}$, which are 20 yards.

28. Sol.—One is contained in $\frac{3}{4}$ $\frac{3}{4}$ of a time; and since one is contained in $\frac{3}{4}$ $\frac{3}{4}$ of a time, $\frac{1}{3}$ is contained in $\frac{3}{4}$ 3 times $\frac{3}{4}$, which are $\frac{9}{4}$ of a time, and $\frac{2}{3}$ is contained in $\frac{3}{4}$ $\frac{1}{2}$ of $\frac{9}{4}$, which is $\frac{9}{8}$ of a time.

49. Ans.—Multiply the dividend by the fraction obtained by inverting the terms of the divisor.

Note.—Assign a few of the problems just analyzed, and require the class to solve them by the Method.

50. Sol.—If a yard of muslin cost $\frac{3}{5}$ of a dime, for $\frac{7}{8}$ of a dime you can purchase as many yards as $\frac{3}{5}$ is contained times in $\frac{7}{8}$, which is $\frac{7}{8} \times \frac{5}{3} = \frac{35}{24}$, or $1\frac{11}{24}$.

LESSON X.

16. Sol.—$\frac{1}{3}$ is $\frac{1}{2}$ of $\frac{2}{3}$, and $\frac{3}{3}$, or one, is 3 times $\frac{1}{2}$, which are $\frac{3}{2}$ of $\frac{2}{3}$. If one is $\frac{3}{2}$ of $\frac{2}{3}$, $\frac{1}{5}$ is $\frac{1}{5}$ of $\frac{3}{2}$, which is $\frac{3}{10}$ of $\frac{2}{3}$, and $\frac{4}{5}$ is 4 times $\frac{3}{10} = \frac{12}{10}$, or $\frac{6}{5}$ of $\frac{2}{3}$. Therefore, $\frac{4}{5}$ is $\frac{6}{5}$ of $\frac{2}{3}$.

Note.—This problem may also be solved by reducing the fractions to a common denominator, thus:

16. Sol.—$\frac{2}{3} = \frac{10}{15}$, and $\frac{4}{5} = \frac{12}{15}$, and $\frac{12}{15}$ is the same part of $\frac{10}{15}$ that 12 is of 10; and 12 is $\frac{12}{10}$, or $\frac{6}{5}$ of 10. Therefore, $\frac{4}{5}$ is $\frac{6}{5}$ of $\frac{2}{3}$.

LESSON XI.

24. Sol.—There were as many persons as $\frac{4}{5}$ is contained times in 8, which are 10, and since he *shared* them, these included himself; hence there were 9 companions.

26. Sug.—$8\frac{1}{3} = \frac{25}{3}$. If $\frac{25}{3}$ peaches are worth 5 apples, $\frac{1}{3}$ of a peach is worth $\frac{1}{25}$ of 5 apples, or $\frac{1}{5}$ of an apple, and $\frac{3}{3}$ of a peach is worth 3 times $\frac{1}{5}$, which are $\frac{3}{5}$ of an apple, and 10 peaches are worth 10 times $\frac{3}{5}$ of an apple, which are 6 apples.

LESSON XII.

1. Sol.—If \$40 was $\frac{2}{3}$ of what remained, $\frac{1}{3}$ of what remained was $\frac{1}{4}$ of \$40, which is \$10, and $\frac{4}{4}$ of what remained was 4 times 10, or \$40. After spending $\frac{3}{4}$ of his fortune, there remained $\frac{4}{4} - \frac{3}{4}$, or $\frac{1}{4}$ of his fortune, which equals \$40, &c.

6. Sug.—If he sold $\frac{1}{2}$ of his sheep, $\frac{1}{2}$ of his sheep remained, which equals 15; hence he owned twice 15, or 30. If $\frac{1}{3}$ of his cows remained, he sold $\frac{2}{3}$ of them, which equals 10; hence he had at first 15 cows.

12. Sug.—He found 6 cents, hence he had 12 — 6, or 6 cents less than he had at first; but he had only $\frac{3}{4}$ as much as at first, hence 6 cents equals $\frac{1}{4}$ of what he had at first; therefore he had at first 4 times 6 cents, or 24 cents.

19. Sug.—She had at first 50, and then had 10; hence she gave away 50 cents — 10 cents, or 40 cents.

Note.—Let the pupils be drilled upon the demonstrations of the Propositions which are given at the close of this lesson. With young pupils use special numbers instead of (n), until they can readily generalize, after which they may give the general demonstration. For valuable exercises of this character, see the "Normal Primary Arithmetic," pages 71, 72, 73, and 74.

SECTION IV.

Note.—After the solution and remarks given for the preceding sections, it will not be necessary to solve any of the problems in this section. It consists principally of the application of the previous principles to denominate numbers. The teacher will find that the remarks upon the tables may be made the means of eliciting much interest in class.

In the solution of such problems as the 8th under Federal Money, the two numbers compared must of course be reduced to the same denomination.

The 5th problem under Apothecaries' Weight, and the 9th under Avoirdupois, are intended to illustrate the relation of the pound and ounce in the three different weights. A pound of lead is absolutely heavier than a pound of gold, since the pound Avoirdupois consists of 7000 grains, and the pound Troy of but 5760. An ounce of silver, however, is heavier than an ounce of feathers, since the Troy ounce is heavier than the Avoirdupois ounce, as may be seen by a very simple reduction, that is, by ascertaining the number of grains in an ounce of each weight. Thus, in Avoirdupois weight there are 16 oz. in a pound, hence 1 oz. $= \frac{1}{16}$ of 7000 grs. $= 437\frac{1}{2}$; but in a pound Troy there are 12 oz., hence 1 oz. $= \frac{1}{12}$ of $5760 = 480$; hence an ounce Troy is $42\frac{1}{2}$ grs. heavier than an ounce Avoirdupois.

SECTION V.

LESSON I.

In the solution of the problems of this lesson let the pupils observe the following suggestion. When we reduce a collection to the unit, pass immediately from this unit to the other collection of the same unit. Thus, if we commence with a collection of horses, and reduce to one horse, pass to the other collection of horses, instead of first reducing some of the other numbers to the unit, and then coming back to the horses.

Many of these problems can be solved without passing to the unit, by employing the relations of the numbers, as is illustrated in Lessons X. and XI., Section II.

9. Sol.—If 10 oxen can eat 4 acres of grass in 6 days, one ox can eat 4 acres of grass in 10 times 6 days, which are 60 days, and 30 oxen can eat 4 acres of grass in $\frac{1}{30}$ of 60 days, which is 2 days; if 30 oxen eat 4 acres of grass in 2 days, they will eat 1 acre in $\frac{1}{4}$ of 2 days, which is $\frac{1}{2}$ of a day, and they will eat 8 acres in 8 times $\frac{1}{2}$, which are $\frac{8}{2}$, or 4 days.

9. Sol. 2d.—If 10 oxen eat 4 acres of grass in 6 days, 30 oxen, which are 3 times 10 oxen, can eat 4 acres of grass in $\frac{1}{3}$ of 6 days, which is 2 days; and if they can eat 4 acres of grass in 2 days, they will eat 8 acres, which are 2 times 4 acres, in 2 times 2 days, which are 4 days.

10. Sug.—Find how many men can perform the same piece in 6 days, and then to perform a piece 3 times as large will require 3 times as many days.

16. Sol.—If 3 oranges are worth 9 cents, one orange is worth $\frac{1}{3}$ of 9 cents, which is 3 cents, and 8 oranges, or 4 melons, are worth 8 times 3 cents, which are 24 cents; if 4 melons are worth 24 cents, one melon is worth $\frac{1}{4}$ of 24 cents, or 6 cents, and 10 melons are worth 10 times 6 cents, which are 60 cents.

Note.—This problem can also be solved by commencing with the melons, thus:

16. Sol.—If 4 melons are worth 8 oranges, one melon is worth $\frac{1}{4}$ of 8 oranges, which is 2 oranges, and 10 melons are worth 10 times 2 oranges, which are 20 oranges; if 3 oranges are worth 9 cents, one orange is worth $\frac{1}{3}$ of 9 cents, which is 3 cents, and 20 oranges are worth 20 times 3 cents, which are 60 cents.

23. Sol.—If A can do as much work in 2 days as B can in 4 days, or C in 6 days, then B can do as much in 4 days as C can in 6 days, and B can do as much in 3 times 4 days, or 12 days, as C can in 18 days, which are 3 times 6 days.

25. Sol.—If A can do 3 times as much in a day as B, and B can do twice as much in a day as C, then A can

do 3 times twice as much as C, which are 6 times as much as C; hence A can do as much in one day as C does in 6 days, and A can do as much in $\frac{1}{6}$ of a day as C can in one day, and A can do as much in $\frac{4}{6}$, or $\frac{2}{3}$ of a day, as C can in 4 days.

27. Sug.—Find how long it will take 8 boys to do $\frac{1}{2}$ of it, and then how long it will take 8 — 3, or 5 boys, to do the other half.

28. Sol.—If 9 men build 10 rods of wall in 8 days, they can build 20 rods, which are 2 times 10 rods, in 2 times 8 days, which are 16 days, and they can build $\frac{1}{4}$ of it in $\frac{1}{4}$ of 16 days, which is 4 days, and $\frac{3}{4}$ of it, what remains, in 3 times 4 days, or 12 days; if 9 men can build $\frac{3}{4}$ of the wall in 12 days, $\frac{1}{3}$ of the number, which remains when $\frac{2}{3}$ leave, can build it in 3 times 12 days, which are 36 days, and the 20 rods will be built in 4 + 36, or 40 days.

29. Sol.—If a measure of flour make 5 four-cent loaves, it will make 5 times 4, or 20 one-cent loaves; and if it will make 20 one-cent loaves, it will make $\frac{1}{2}$ of 20, or 10 two-cent loaves.

LESSON II.

2. Sug.—After finding the number of pupils to be 13, we find the number of questions by multiplying 2 by 13, and then adding 26, or by multiplying 4 by 13.

7. Sol.—The difference between having 10 more and 30 more, is 20, and the difference between having 2 times as many as Robert and 4 times as many as Robert, is twice as many as Robert; hence twice Robert's number equals 20, and Robert's number equals $\frac{1}{2}$ of 20, or 10, &c.

7. Sol. 2d.—By the first condition, twice Robert's equals Morris' plus 10; by the second condition, 4 times Robert's equals Morris' plus 30: hence 4 times, minus 2 times, or 2 times Robert's equals 30 — 10, or 20, &c.

10. Sug.—We find the cost of one orange is 4 cents, and the cost of one apple is 2 cents, the difference between the cost of each is 2 cents, the difference between the cost of all is 18 cents; hence there were as many of each as 2 is contained times in 18, which are 9.

12. Sol.—The difference between having 8 more and 12 less is 20, and the difference between having 6 times as many as Henry, and 2 times as many as Henry, is 4 times as many as Henry; hence 4 times Henry's number equals 20, &c.

13. Sol.—If one girl received 3 apples, and one boy received 4 apples, one girl and one boy received 3 apples plus 4 apples, which are 7 apples; and since there were the same number of each, and all received 28 apples, there were as many of each as 7 is contained times in 28, which are 4.

17. Sol.—If one boy receives 2 cents, 3 boys will receive 3 times 2 cents, which are 6 cents, and one girl and 3 boys will receive 4 cents plus 6 cents, which are 10 cents, and they all received 60 cents; hence there were as many times one girl and 3 boys as 10 is contained times in 60, which are 6; hence there were 6 girls, and 6 times 3, or 18 boys.

18. Sug.—We find the difference between the cost of 2 apples and one orange is 2 cents, and the difference between the cost of all is 10 cents; hence there were as many times one orange and 2 apples as 2 is contained times in 10, which are 5; hence there were 5 oranges, and 5 times 2, which are 10 apples.

LESSON III.

6. Sol.—Three times a certain number equals $\frac{9}{3}$ of the number, which increased by $\frac{2}{3}$ of the number equals $\frac{11}{3}$ of the number, which by a condition of the problem equals 22, &c.

12. Sug.—We find that $\frac{5}{3}$ of the height, plus 10 feet, equals $\frac{6}{3}$ of the height; then $\frac{6}{3}$ of the height, minus $\frac{5}{3}$ of the height, which is $\frac{1}{3}$ of the height, equals 10 feet, &c.

16. Sol.—If 2 times a number, plus 6, equals 3 times the same number, plus 2, then 3 times the number, minus 2 times the number, which is once the number, equals 6 minus 2, which is 4.

19. Sol.—If 4 times A's age, minus 10 years, equals 3 times A's age, plus 10 years, 4 times A's age equals 3 times A's age, plus 20 years; and if 4 times A's age equals 3 times A's age, plus 20 years, 4 times A's age, minus 3 times A's age, which is once A's age, equals 20 years.

19. Sol. 2d.—If 4 times A's age, minus 10 years, equals 3 times A's age, plus 10 years, 4 times minus 3 times, or once A's age $= 10 + 10$, or 20 years.

23. Sug.—After spending $\frac{3}{4}$ of what he borrowed, there remained $\frac{1}{4}$ of what he borrowed, that is, $\frac{1}{4}$ of $\frac{2}{3}$, which is $\frac{1}{6}$ of Emily's money, which is $20, &c.

24. Sug.—If the thief spent $\frac{2}{3}$ of what he stole, there remained $\frac{1}{3}$ of what he stole, that is, $\frac{1}{3}$ of $\frac{3}{5}$ of Harry's money, which is $\frac{1}{5}$ of Harry's; then $\frac{3}{5}$ of Harry's money, what was stolen, minus $\frac{1}{5}$ of his money, what was given back, equals $\frac{2}{5}$ of Harry's money, which equals $20, &c.

25. Sug.—Two times a number plus 10, equals 3 times (the number plus 2), which is 3 times the number plus 6; then 3 times the number, minus 2 times the number, which is once the number, equals 10 minus 6, or 4, &c.

26. Sug.—If the thief had spent $\frac{4}{5}$ of what he stole, there remained $\frac{1}{5}$ of what he stole, that is, $\frac{1}{5}$ of $\frac{5}{7}$ of Baldwin's money, which is $\frac{1}{7}$ of his money; then $\frac{2}{7}$ of his money, what Baldwin had remaining, minus $\frac{1}{7}$ of his money, which is $\frac{1}{7}$ of his money, equals $30, &c.

LESSON IV.

7. SOL.—If $\frac{1}{3}$ of the longer piece equals the shorter, then $\frac{3}{3}$ of the longer piece, plus $\frac{1}{3}$ of the longer piece, which is the shorter piece, equals $\frac{4}{3}$ of the longer piece, which, by the condition of the problem, equals 36 feet. If $\frac{4}{3}$ of the longer equals 36 feet, $\frac{1}{3}$ of the longer, which is the shorter, equals $\frac{1}{4}$ of 36 feet, which is 9 feet, &c.

11. SUG.—We find the sum of the numbers equals 15. If 3 times the smaller number equals twice the greater, once the smaller equals $\frac{1}{3}$ of twice the greater, which is $\frac{2}{3}$ of the greater; then $\frac{2}{3}$ of the greater number, which is the smaller, plus $\frac{3}{3}$ of the greater, which is $\frac{5}{3}$ of the greater number, equals 15, &c.

14. SOL.—Twice what the first has equals what the second has, and 3 times what the first has equals what the third has; then once what the first has, plus twice what the first has, which is what the second has, plus 3 times what the first has, which is what the third has, equals 6 times what the first has, which equals 36 apples, &c.

17. SOL.—If B earned twice as much as C, then twice what C earned equals what B earned, and if A earned twice as much as B, 2 times twice what C earned, which is 4 times what C earned, equals what A earned, &c.

21. SUG.—The difference between 4 times the smaller and once the smaller, which is 3 times the smaller, equals the difference between the two numbers, which is 27.

25. SOL.—If there are $\frac{3}{4}$ as many sheep as hogs, $\frac{3}{4}$ of the number of hogs equals the number of sheep; and if there are $\frac{4}{5}$ as many cows as hogs, $\frac{4}{5}$ of the number of hogs equals the number of cows, &c.

26. SOL.—Since $\frac{3}{4}$ of the length in the water equals the length in the mud, $\frac{4}{4}$ of the length in the water, minus $\frac{3}{4}$ of the length in the water, which is $\frac{1}{4}$ of the

length in the water, equals 10 feet; $\frac{3}{4}$ of the length in the water, which is the length in the mud $=$ 3 times 10 feet, or 30 feet, and $\frac{4}{4}$ of the length in the water $= 4 \times 10 = 40$ feet, which is $\frac{2}{3}$ of the length in the air, &c.

LESSON V.

5. SUG.—Before Reuben found \$9, they had \$45 — \$9 = \$36; hence each at first had $\frac{1}{2}$ of \$36, which is \$18.

8. SUG.—They both lost 10 cents; hence before losing any they had 22 cents plus 10 cents, which are 32 cents; and each, therefore, found $\frac{1}{2}$ of 32 cents, which is 16 cents.

11. SOL.—By a condition of the problem, 3 times Harry's, plus 5 years, equals Harvey's age, which, added to Harry's age, equals 4 times Harry's, plus 5 years, which is 45 years. If 4 times Harry's age, plus 5 years, equals 45 years, 4 times Harry's age equals 45 years, minus 5 years, which is 40 years, &c.

15. SOL.—Three fifths of the larger piece, plus 5 feet, equals the shorter, which, added to the larger, equals $\frac{8}{5}$ of the larger, plus 5 feet, which equals 45 feet, &c.

19. SOL.—If he walked 5 miles further the second day than the first, and 10 miles further the third day than the second, then once the distance he walked the first day, plus 5 miles, equals the distance he walked the second day, and once the distance he walked the first day, plus 5 miles, plus 10 miles, which is once the distance he walked the first day, plus 15 miles, equals the distance he walked the third day, &c.

21. SUG.—We find that 7 times the cost of the hat, plus \$8, equals \$78; hence 7 times the cost of the hat equals \$78 — \$8 = \$70, &c.

23. SUG.—A earned $\frac{2}{3}$ of $\frac{3}{4}$ as much as C, which is $\frac{2}{4}$ as much as C; then $\frac{2}{4}$ of what C earned, which is what A earned, plus $\frac{3}{4}$ of what C earned, which is what B earned, plus $\frac{4}{4}$ of what C earned, which is what C earned, equals $\frac{9}{4}$ of what C earned, which is \$108. If $\frac{9}{4}$ of what C earned equals \$108, $\frac{1}{4}$ of what C earned equals $\frac{1}{9}$ of \$108, which is \$12, &c.

24. SUG.—$\frac{3}{6}$ of the number of sheep, plus 10, which is the number of cows, plus $\frac{2}{6}$ of the number of sheep, plus 10, which is the number of horses, plus $\frac{6}{6}$ of the number of sheep, equals $\frac{11}{6}$ of the number of sheep, plus 20, which equals 42; hence $\frac{11}{6}$ of the number of sheep equals $42 - 20 = 22$, &c.

SECTION VI.

IT will be noticed that the Section on Per Cent. and Interest is not placed near the latter part of the book, as is the custom of most authors. There are two reasons for this; first, these subjects are much easier than some which follow them, and secondly, being of a more practical character, they should be inserted as soon as the pupil is prepared for them.

It will be found, also, that the subject of per cent. is more thoroughly treated than in any previous work of this kind. A large collection of interesting problems, involving novel and useful combinations, has been introduced, by which original thought on the part of the pupil will be secured.

ERRORS TO BE AVOIDED.—In the solutions of problems in this section, teachers should be particular that the words *per cent.* and *cents* are not used synonymously. Learners, somehow, get the idea that per cent. has some very intimate connection with cents, and, unless guarded, will use the one for the other.

The expression, "5 per cent. on a hundred," is frequently used by learners. This, of course, is equivalent to "5 on a hundred on a hundred," the absurdity of which is evident.

Another error is the expression, "25 per cent. equals $\frac{1}{4}$." This is not exactly true. That 25 per cent. of anything is equal to $\frac{1}{4}$ of that thing, is true; but it is not true that "25 on a hundred equals $\frac{1}{4}$."

LESSON I.

2. Rem.—This is designed for four different problems, the object of the author being to economize space.

7. Sol.—$8\frac{1}{3}$ per cent. equals $\frac{25}{3}$ per cent. A gain of $\frac{25}{3}$ per cent. is a gain of $\frac{25}{3}$ on 100. If on 100 the gain is $\frac{25}{3}$, on 1 it is $\frac{1}{100}$ of $\frac{25}{3}$, which is $\frac{25}{300}$, or $\frac{1}{12}$. Therefore, at a gain of $8\frac{1}{3}$ per cent., $\frac{1}{12}$ of the cost equals the gain.

12. Sug.—Find 20 per cent. of \$50, and then find 20 per cent. of the remainder, and subtract; or, find 80 per cent. of \$50, and then 80 per cent. of this.

16. Sug.—Each yard cost 30 cents; hence the gain on each yard was $\frac{10}{100}$, or $\frac{1}{10}$ of 30 cents.

LESSON II.

6. Sug.—We find he lost $\frac{1}{3}$ of the cost, but the cost was 100 per cent. of itself; hence he lost $\frac{1}{3}$ of 100 per cent., which was $33\frac{1}{3}$ per cent.

Rem.—It will be observed that we do not use the price of the horse in the solution; there are several problems of this character in the work, the object being to exercise the judgment of the pupil as to the number of *essential* conditions.

8. SUG.—This problem usually gives rise to considerable discussion, some obtaining 20, and others 25 per cent., for the answer. The difference arises in computing the per centage upon the cost of 10 cows, or upon the cost of 8 cows, the number sold. If the rate per cent. is reckoned upon 10, the result is 20, if upon 8 it is 25. It is evident that the latter is correct, since a merchant reckons the gain of a sale, not upon the value of his whole stock, but upon the value of the goods sold.

9. SOL.—\$25 is 100 per cent. of \$25; and since \$25 is 100 per cent. of \$25, \$5, which is $\frac{1}{5}$ of \$25, is $\frac{1}{5}$ of 100 per cent., which is 20 per cent of \$25.

12. SUG.—After losing 20 per cent. there remained 80 per cent.; then selling 25 per cent., there remained $\frac{75}{100}$, or $\frac{3}{4}$ of 80 per cent., which is 60 per cent.

13. SUG.—Here are several distinct problems. We will solve the first and last.

SOL. *of 1st.*—$\frac{1}{4}$ is 100 per cent. of $\frac{1}{4}$; and since $\frac{1}{4}$ is 100 per cent. of $\frac{1}{4}$, $\frac{1}{8}$, which is $\frac{1}{2}$ of $\frac{1}{4}$, is $\frac{1}{2}$ of 100 per cent., which is 50 per cent. of $\frac{1}{4}$.

SOL. *of the last.*—$\frac{2}{3}$ is 100 per cent. of $\frac{2}{3}$; hence $\frac{1}{3}$ is $\frac{1}{2}$ of 100 per cent., which is 50 per cent. of $\frac{2}{3}$, and $\frac{3}{3}$, or 1, is 3 times 50 per cent., which are 150 per cent. of $\frac{2}{3}$; and $\frac{1}{5}$ is $\frac{1}{5}$ of 150 per cent. of $\frac{2}{3}$, which is 30 per cent. of $\frac{2}{3}$, and $\frac{3}{5}$ is 3 times 30 per cent., which are 90 per cent. of $\frac{2}{3}$.

15. SUG.—$\frac{2}{3}$ of \$6 = \$4; $\frac{4}{5}$ of \$50 equals \$40. If \$4 is twice some per cent. of \$40, once that per cent. of \$40 equals $\frac{1}{2}$ of \$4, which is \$2, and \$2 is 5 per cent. of \$40. Therefore, $\frac{2}{3}$ of \$6 is twice 5 per cent. of $\frac{4}{5}$ of \$50.

24. SUG.—\$12 equals $\frac{6}{5}$ of the cost, from which we can find the cost and the loss if sold for \$8.

LESSON III.

7. SUG.—Find the cost as in problem (1), and then what he would have received, at a gain of 25 per cent.

12. SUG.—Find the value of the horse and carriage, then the gain if sold for $300, and then the gain per cent.

14. SOL.—If on the first he gained 25 per cent., then $\frac{25}{100}$, or $\frac{1}{4}$ of the cost, equals the gain, which, added to $\frac{4}{4}$ of the cost, equals $\frac{5}{4}$ of the cost, which equals $15; if $\frac{5}{4}$ of the cost equals $15, $\frac{1}{4}$ of the cost, which is the gain, equals $\frac{1}{5}$ of $15, which is $3. If on the second book he lost 25 per cent., then $\frac{25}{100}$, or $\frac{1}{4}$ of the cost, equals the loss, which, subtracted from $\frac{4}{4}$ of the cost, equals $\frac{3}{4}$ of the cost, which equals $15; if $\frac{3}{4}$ of the cost equals $15, $\frac{1}{4}$ of the cost, which is the loss, equals $\frac{1}{3}$ of $15, which is $5. Since on one he gained $3, and on the other he lost $5, he lost $5 minus $3, which is $2, by the transaction.

NOTE.—The 14th problem is sometimes solved by finding the cost of both books, then the gain on the first and loss on the second, and then the difference of these. This solution is longer than the one given, and no clearer.

18. REM.—This problem usually gives rise to considerable discussion; some hold that, on the stove bought, he gained $10, thus considering $30 $\frac{3}{4}$ of the value, and hence estimating the gain upon the *value* of the stove; others say that the gain on the stove bought is 7\frac{1}{2}$, that is, 25 per cent. of $30, thus estimating the gain upon the money expended. The latter view is undoubtedly the correct one, since in a business transaction gain is always estimated upon the money employed in the purchase, and not upon the value of the article. In this way both the buyer and seller may gain.

19. SUG.—We find $\frac{3}{3}$ of what she received for the painting equals $\frac{6}{5}$ of the cost; hence she gained $\frac{1}{5}$ of the cost, or 20 per cent.

LESSON IV.

1. Sug.—\$20 equals $\frac{25}{100}$, or $\frac{1}{4}$ of the cost; hence the cost is \$80.

5. Sol.—If 4 is 10 per cent. of some number, then 4 is $\frac{10}{100}$, or $\frac{1}{10}$ of some number, and $\frac{10}{10}$ of that number is 10 times 4, which are 40.

7. Sol.—If 30 is 25 per cent. less than some number, then 30 is $\frac{25}{100}$, or $\frac{1}{4}$ less than some number; hence 30 is $\frac{3}{4}$ of some number, &c.

8. Sol.—If he gained \$20 by selling for 20 per cent. above its value, by selling for 10 per cent. above its value, which is $\frac{1}{2}$ of 20 per cent., he would have gained $\frac{1}{2}$ of \$20, which is \$10.

16. Sug.—If A's money is $\frac{25}{100}$, or $\frac{1}{4}$ more than B's, then $\frac{5}{4}$ of B's money equals A's money, and the difference between A's money and B's money is $\frac{1}{4}$ of B's money; and since $\frac{5}{4}$ of B's money equals A's money, this difference is $\frac{1}{5}$ of A's, or 20 per cent. of A's money.

21. Sug.—\$8 is 20 per cent., or $\frac{1}{5}$ of the cost; hence the cost is \$40, and therefore he can buy 10 yards.

LESSON V.

These problems are so easy that it is not necessary to give the solution of many of them. We solve the 21st as a model for those which follow.

21. Sol.—At 5 per cent. $\frac{5}{100}$ of the principal equals the interest for one year, and for 6 years 6 times $\frac{5}{100}$, which are $\frac{30}{100}$, or $\frac{3}{10}$ of the principal equals the interest; $\frac{1}{10}$ of \$60 is \$6, and $\frac{3}{10}$ of \$60 equals 3 times \$6, which are \$18.

LESSON VI.

In finding the amount, the pupil may add the interest to the principal, or he may obtain it by the method given in the solution in the Mental Arithmetic. We prefer the latter method, since it prepares the pupil to find the principal when the amount, rate per cent., and time are given.

When the pupils are familiar with the method of finding the part of the principal which equals the interest, as given in Lesson V., page 107, it may be abbreviated, as is indicated in the following solution.

13. SOL.—For 8 years, at 5 per cent., $\frac{40}{100}$, or $\frac{2}{5}$ of the principal equals the interest, which, added to $\frac{5}{5}$ of the principal, equals $\frac{7}{5}$ of the principal, which equals the amount; $\frac{1}{5}$ of \$500 equals \$100, and $\frac{7}{5}$ of \$500 equals 7 times \$100, which are \$700. Now, A's share is 6 times B's, which, added to B's, equals 7 times, which equals \$700, &c.

16. SUG.—We find A's interest equals \$40, and since A's fortune is $\frac{1}{4}$ of B's, A's interest is $\frac{1}{4}$ of B's interest; hence \$40 is $\frac{1}{4}$ of B's interest, and $\frac{4}{4}$ of B's interest is 4 times \$40, or \$160.

17. SUG.—Find C's amount, and this is $\frac{3}{4}$ of D's amount, from which D's amount can be readily obtained.

18. SUG.—Find A's interest, and this is $\frac{2}{3}$ of B's, from which find B's, and then subtract one from the other.

A shorter method is the following: Since A has $\frac{2}{3}$ as much as B, A has $\frac{1}{3}$ of B's more than B, which is $\frac{1}{2}$ of \$400; hence $\frac{2}{5}$ of \$200 = \$80 is the difference.

19. REM.—Some find the principal of each, and then the interest of each principal. It is better, however, to find the interest of \$1200, and then divide this interest according to the conditions of the problem.

LESSON VII.

15. Sug.—We find the principal to be $600, which divided according to the conditions of the problem, gives $200 for A's, and $400 for B's fortune.

17. Sug.—We find the sum of $\frac{1}{2}$ of A's and $\frac{1}{3}$ of B's fortune to be $800; hence, since $\frac{1}{2}$ of A's equals $\frac{1}{3}$ of B's, $\frac{1}{2}$ of $800, or $400, equals $\frac{1}{2}$ of A's, and also $\frac{1}{3}$ of B's fortune, &c.

18. Sug.—This problem is usually solved by finding how much Martin owes, and then how much money is due him, and then taking this difference. The following method, however, is much shorter.

We find that $\frac{1}{5}$ of the principal equals the interest, and the difference of the interest of the money he owes and the money due him, is $30; hence $30 is $\frac{1}{5}$ of what is due above what he owes.

LESSON VIII.

7. Rem.—Pupils in solving these problems usually say, as in the previous examples, $\frac{1}{5}$ of the *principal* equals the *interest*. This is not precise. It should be, $\frac{1}{5}$ of the present worth equals the discount.

13. Sug.—Pupils usually find the present worth, and subtract it from the debt to find the discount. The shorter way is to find the value of the part of the present worth which equals the discount, thus: $\frac{9}{20}$ of the present worth equals the discount, and $\frac{29}{20}$ of the present worth equals $580, $\frac{1}{20}$ of the present worth equals $\frac{1}{29}$ of $580, which is $20, and $\frac{9}{20}$ of the present worth, which is the discount, equals 9 times 20, or $180.

20. Sug.—This means that 3 times B's equals A's; hence 4 times A's plus A's, which is 5 times A's, equals $3000.

22. SUG.—The principal is \$1500. If the house cost $\frac{1}{3}$ as much as the farm, $\frac{2}{3}$ of the cost of the farm equals twice the cost of the house; hence $2\frac{1}{2}$ times, or $\frac{5}{2}$ of the cost of the house equals \$1500, &c.

24. SUG.—We find that $\frac{1}{2}$ of the principal equals the interest, and $\frac{3}{2}$ of the principal equals the amount; hence $\frac{3}{2}$ of first interest equals \$270, and the first interest equals \$180, which is $\frac{1}{2}$ of the principal; hence, the principal equals \$360.

Or, since $\frac{1}{2}$ of the first principal equals the interest, and $\frac{3}{2}$ of this interest equals the amount, $\frac{3}{2}$ of $\frac{1}{2}$, or $\frac{3}{4}$ of the cost of the horse, &c., equals \$270; hence $\frac{4}{4}$ of the cost equals \$360.

LESSON IX.

15. REM.—This problem can also be solved by the following method: A principal will gain twice itself in 200 years, at *one* per cent., and at 40 per cent. in $\frac{1}{40}$ of 200 years, or 5 years.

17. SOL.—At 5 per cent. for *one* year, $\frac{5}{100}$, or $\frac{1}{20}$ of the principal, equals the interest. For a principal to double itself, it must gain once itself, or $\frac{20}{20}$ of itself; hence it will require as many years as $\frac{1}{20}$ is contained times in $\frac{20}{20}$, which are 20.

21. SUG.—\$280, the amount at 8 per cent., minus \$250, the amount at 5 per cent., equals \$30, the *interest* on the given principal for the given time at 8 per cent., minus 5 per cent., or 3 per cent.; hence the interest at one per cent. is $\frac{1}{3}$ of \$30, which is \$10, and 5 per cent. 5 times \$10, or \$50, and the principal is \$250 — \$50, or \$200. The time is found as in the first problem of this lesson.

LESSON X.

13. Rem.—By a little analysis it may be shown that a principal will gain 3 times itself, at 300 per cent., in *one* year, and hence in 10 years, at $\frac{1}{10}$ of 300 per cent., or 30 per cent.

17. Sol.—In 10 years at *one* per cent., $\frac{10}{100}$, or $\frac{1}{10}$ of the principal, equals the interest. For a principal to treble itself it must gain twice itself, or $\frac{20}{10}$ of itself; hence it will require as many times one per cent. as $\frac{1}{10}$ is contained times in $\frac{20}{10}$, which are 20.

23. Sug.—The difference between $600 and $540, which is $60, equals the interest for 10 years minus 7 years, which is 3 years; hence the interest for one year is $20, and for 10 years $200; hence the principal equals $600 — $200 = $400. The rate per cent. is found as in the first problem of this lesson.

SECTION VII.

LESSON I.

5. Sol.—If 10 sheep eat as much as 1 ox, 100 sheep, which are 10 times 10 sheep, will eat as much as 10 oxen; hence 12 oxen and 100 sheep will eat as much as 12 oxen plus 10 oxen, which are 22 oxen. If the pasturage of 22 oxen cost $44, &c.

8. Sol.—If 2 men do as much as 3 boys, 6 men, which are 3 times 2 men, will do as much as 3 times 3 boys, which are 9 boys; hence 6 men and 15 boys will do as much as 9 boys plus 15 boys, which are 24 boys. If 24 boys earn $72, &c.

9. Sug.—Three men for 5 days will earn as much as 15 men for one day, and 4 men 3 days will earn as much as 12 men in one day, &c.

Rem.—Some arithmeticians say, 3 men for 5 days is the *same* as 15 men for 1 day, &c. This is not quite true; "*will earn,*" &c., is better

15. SUG.—Twelve horses, which are 4 times 3 horses, will do as much as 4 times 5, or 20 oxen; then 12 horses and 18 oxen will do as much as 20 oxen plus 18 oxen, or 38 oxen, &c.

18. SUG.—We find a horse eats as much as 2 cows, or as much as 8 sheep; hence 18 cows will eat as much as 9 horses, and 48 sheep will eat as much as 6 horses, and they will all eat as much as 21 horses. If the pasturage of 21 horses is $63, &c.

LESSON II.

1. REM.—In the solution of such problems, some say, "Since the parts are as 4 to 6, we *must* divide," &c. This is not true; we could divide into $2 + 3$, which are 5 equal parts. Instead of saying *must*, say, if we divide into $4 + 6$, &c.

10. SOL.—One-half equals $\frac{3}{6}$ and $\frac{1}{3}$ equals $\frac{2}{6}$; hence the numbers are to each other as $\frac{3}{6}$ is to $\frac{2}{6}$, or as 3 to 2. Therefore, if we divide 50 into two 3 plus 2, which are 5 equal parts, 3 of these parts, or $\frac{3}{5}$ of 50, will be one number, &c.

15. SUG.—The reciprocal of a number equals unity divided by that number. The reciprocal of 2 is $\frac{1}{2}$, and the reciprocal of 3 is $\frac{1}{3}$; hence they pay in the proportion of $\frac{1}{2}$ to $\frac{1}{3}$, or $\frac{3}{6}$ to $\frac{2}{6}$, or 3 to 2, &c.

18. SUG.—This problem may be solved in several different ways. We suggest two of the best.

First. If A's is to B's as 3 to 4, 4 times A's = 3 times B's, or B's = $\frac{4}{3}$ of A's, and $\frac{1}{2}$ of B's = $\frac{2}{3}$ of A's, which, added to A's, equals $\frac{5}{3}$ of A's, which equals $2000, from which A's and B's can readily be found.

Second. If A's is to B's as 3 to 4, A's is to $\frac{1}{2}$ of B's as 3 to 2; hence if we divide $2000 in two parts, which are as 3 to 2, the first part will be A's and the second part $\frac{1}{2}$ of B's, &c.

19. Sug.—After finding the sum of $\frac{1}{3}$ of A's and $\frac{1}{4}$ of B's, we proceed thus: If A's is to B's as 9 to 8, $\frac{1}{3}$ of A's is to $\frac{1}{4}$ of B's as 3 to 2; hence if we divide \$500 into two parts which are to each other as 3 to 2, the first part will be $\frac{1}{3}$ of A's, and the second part $\frac{1}{4}$ of B's, &c.

LESSON III.

18. Sug.—Subtract what A and B do in a day from what A, B, and C do in a day, and we have what C does in a day; subtract this from what B and C do in a day, and we have what B does in a day; subtract this from what A and B do in a day, and we have what A does in one day, from which we can find the time in which each can do it.

23. Sug.—We find that E can build the fence in 18 days; hence he can build $\frac{1}{3}$ of it, which remains after D has built $\frac{2}{3}$ of it, in $\frac{1}{3}$ of 18 days, or 6 days.

24. Sol.—If 2 men can plough an acre in $\frac{1}{6}$ of a day, they can plough 6 acres in one day, one man can plough $\frac{1}{2}$ of 6 acres, or 3 acres, and 3 men can plough 9 acres in a day. If 3 boys can plough 6 acres, one boy can plough 2 acres, and 2 boys can plough 4 acres; hence 3 men and 2 boys can plough 13 acres in a day, and they can plough 1 acre in $\frac{1}{13}$ of a day.

26. Sug.—A, B, and C did $\frac{1}{3}$ of it; hence there remained $\frac{2}{3}$ of it. A and B can do the whole in 9 days; hence they can do $\frac{2}{3}$ of it in $\frac{2}{3}$ of 9 days, or 6 days.

27. Sug.—Sallie can make it in 3 days, and Euretta in 4 days; hence the three will make $\frac{1}{6}+\frac{1}{4}+\frac{1}{3}=\frac{9}{12}$ of it in a day, and after working $\frac{2}{3}$ of a day there remained $\frac{1}{2}$ of it to be made. Marie and Sallie can make $\frac{1}{6}+\frac{1}{3}=\frac{1}{2}$ of it in one day; hence it will require one day for Marie and Sallie to finish it.

LESSON IV.

9. Sol.—If $\frac{3}{4}$ of C's age equals $\frac{2}{3}$ of B's, $+$ 6, $\frac{1}{4}$ of C's equals $\frac{2}{9}$ of C's, $+$ 2, and $\frac{4}{4}$ of C's equals $\frac{8}{9}$ of C's, $+$ 8, which, added to $\frac{9}{9}$ of C's, equals $\frac{17}{9}$ of C's, $+$ 8, which equals 34, &c.

11. Sug.—We find that $\frac{5}{5}$ of Fanny's number equals $\frac{15}{8}$ of Sallie's number; then $\frac{15}{8}$ of Sallie's number, minus $\frac{8}{8}$ of Sallie's number, which is $\frac{7}{8}$ of Sallie's number, equals 14, &c.

15. Sug.—We find that 3 times what A builds equals what they both build, or $\frac{1}{6}$ of a boat; then once what A builds equals $\frac{1}{3}$ of $\frac{1}{6}$ of a boat, or $\frac{1}{18}$ of the boat, and twice what A builds, or what B builds, equals $\frac{1}{9}$ of the boat; hence it will require A 18 days, and B 9 days to build the boat.

19. Sug.—They can build $\frac{1}{6}$ in one day, which, divided into three parts, which are as 1, 2, and 3, gives $\frac{1}{36}$, $\frac{1}{18}$, and $\frac{1}{12}$, the respective parts each can do in one day; hence it will take A 36 days, B 18 days, and C 12 days.

22. Sug.—B can drink $\frac{1}{4}$ of it in a day, A $\frac{1}{3}$ of it, and C $\frac{1}{2}$ of it in a day; A, B, and C can drink $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{13}{12}$ of it in a day; hence after drinking $\frac{6}{13}$ of a day there will remain $\frac{1}{2}$ of it for A and C to drink. A and C can drink $\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$ of it in a day, and to drink $\frac{1}{2}$ of it it will take them $\frac{3}{5}$ of a day.

LESSON V.

2. Sug.—He can save \$2 each day, and in 40 days he can save \$80, hence he loses \$80 — \$50 = \$30; each day he is idle he loses \$2$\frac{1}{2}$, hence to lose \$30 it will require 12 days.

Note.—Some obtain 10 days for the result, assuming that he lost \$3 each day he was idle; but this is incorrect, as may be seen by an attempt to prove the problem. After paying his board for the whole time, which we assume in the first part of the solution, he can lose, by being idle, only his daily wages.

13. SOL.—If he received \$2 a day, and agreed to labor for \$60, he agreed to labor as many days as \$2 is contained times in \$60, which are 30, &c.

17. SUG.—We find he can save \$16 in a week, and in 10 weeks he can save \$160; hence he loses \$160 — \$144 = \$16. Now, in computing what he loses, we have regarded his board as paid for the whole time, hence each day he is idle he loses \$4; therefore, to lose \$16, he must idle 4 days, &c.

19. SUG.—By the first condition A pays $\frac{2}{5}$ and B $\frac{3}{5}$. hence $\frac{2}{3}$ of what B pays equals what A pays. By the second condition B pays $\frac{2}{5}$, and C $\frac{3}{5}$, hence $\frac{2}{3}$ of what C pays equals what B pays, and therefore A pays $\frac{2}{3}$ of $\frac{2}{3}$ of what C pays, or $\frac{4}{9}$ of what C pays; then $\frac{9}{9}$ of what C pays minus $\frac{4}{9}$ of what C pays, which is $\frac{5}{9}$ of what C pays, equals \$500, &c.

LESSON VI.

It will be seen that the problems in the first part of this lesson are of a more general character than those usually given by arithmeticians. As usually stated, the requirement is, when will one be *twice* as old as the other, and the solution given is of a special nature, and cannot be applied to the general problem; that is, when the requirements are, when will one be *three* times, or *four* times, or *five* times, &c., as old as the other. By the solution here given we cannot only solve the general problem, when *future* time is involved, but also when *past* time is considered.

9. SUG.—First find the age of one, as John, and then proceed as before.

11. SUG.—Find the age of one of them, as in Lesson IV., and then proceed as before.

16. SUG.—He paid 6 cents for 2 oranges, and sold 2 for 8 cents, and thus gained 2 cents on 2 oranges; hence, to gain 12 cents, which are 6 times 2 cents, he sold 6 times 2 oranges, or 12 oranges, &c.

19. SUG.—If 3 yards cost \$1, 1 yard cost $\$\frac{1}{3}$, and if 4 yards cost \$1, 1 yard cost $\$\frac{1}{4}$; hence 2 yards cost $\$\frac{1}{3} + \$\frac{1}{4} = \$\frac{7}{12}$: he sold 8 yards for \$3, hence he sold 2 yards for $\$\frac{9}{12}$, and therefore gained $\$\frac{2}{12}$, or $\$\frac{1}{6}$, on 2 yards; hence to gain \$5 he must sell 60 yards, &c.

23. SOL.—As often as he paid 2 cents for 1 apple he paid 8 cents for 2 oranges, hence 1 apple and 2 oranges cost 10 cents, and 3 were sold for 9 cents; hence he lost on 1 apple and 2 oranges 1 cent, and to lose 100 cents he must sell 100 times 1 apple and 2 oranges, which are 100 apples and 200 oranges.

LESSON VII.

The problems in the first part of this lesson are usually found by pupils to be rather difficult; we propose, therefore, to present a solution of a few in a more analytical form than given in the text-book.

2. SOL.—5 times O's age now = John's age now, and
5 times O's age now + 8 yrs. = John's age in 8 yrs., and
1 time O's age now + 8 yrs. = Oliver's age in 8 yrs., then
5 times O's age now + 8 yrs. = 3 times (once O's now + 8 yrs.), or
5 times O's age now + 8 yrs. = 3 times O's now + 24 yrs., hence
5 times O's age now — 3 times O's age now = 24 yrs. minus 8 yrs., or
2 times O's age now = 16 yrs.,
1 time O's age now = 8 yrs., and
5 times O's now, or John's age = 5 × 8 yrs. = 40 yrs.

3. Sol.—4 M's age now = Aunt's age now,
4 M's age now + 20 yrs. = Aunt's age in 20 yrs.,
1 M's age now + 20 years = Aunt's age in 20 years, then
4 M's age now + 20 yrs. = 2 (M's age now + 20 yrs.), or
4 M's age now + 20 yrs. = 2 M's age now + 40 yrs., hence
4 M's age now — 2 M's age now = 40 yrs. — 20 yrs., or
2 M's age now = 20 yrs.,
M's age now = 10 yrs.,
4 M's age now = 40 yrs.

7. Sug.—The difference between 25 years ago and 5 years ago is 20 years; hence 7 times Willard's age 25 years ago plus 20 years = his uncle's age 5 years ago, &c.

14. Sol.—Since 2 of the hound's leaps equal 8 of the hare's, for the hare to run as fast as the hound, it must take 8 leaps while the hound takes 2 leaps, but, by the problem, the hare takes only 4 while the hound takes 2, therefore the hound gains 8 — 4, or 4 of the hare's leaps in taking 2 leaps; hence to gain 10 leaps he must take $\frac{1}{4}$ of 2, or $\frac{1}{2}$ of a leap, and to gain 30 leaps, the distance the hare is ahead, he must take 30 times $\frac{1}{2}$ of a leap, which are 15 leaps.

16. Sug.—We find that the thief loses 2 steps in taking 6 steps; hence to lose 20 steps, which are 10 times 2 steps, he must take 10 times 6 steps, or 60 steps.

18. Sug.—We find that when B takes 6 steps A will take 3 steps, and since 6 of B's = 4 of A's, A will lose 1 step every time he takes 3 steps and B takes 6; hence to lose 10 steps, the distance he was ahead, A will take 10 times 3, or 30 steps, and B will take 10 times 6, or 60 steps.

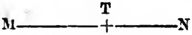
19. Sug.—We find that B, in taking 2 steps, gains 2 of C's steps; hence in taking 30 steps he will gain 30 of C's steps, and therefore C was 30 steps ahead when they started.

22. SUG.—We find that 6 of B's = 3 of A's; hence when B takes 9 steps A goes a distance equal to 6 of B's steps, from which we can find the distance in B's steps each goes before they meet, and reducing the distance A goes in B's steps to A's steps, we have the number of steps each takes before they are together.

LESSON VIII.

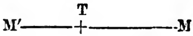
The "time problems" in this lesson are usually regarded by pupils as being among the most difficult in the book. If the time be represented by a line, these may be made as simple as the problems in Lesson III., Section V. It will be well for the teacher to solve a few of them by the method suggested below, and also require the class to indicate them upon the blackboard.

2. SUG.—The line M N represents the time from midnight to noon, and M T present time. Then $\frac{2}{3}$ of M T = T N, which, added to $\frac{3}{3}$ of M T = $\frac{5}{3}$ of M T, which equals M N, or 12 hours, and $\frac{3}{3}$ of M T = $\frac{36}{5}$ = $7\frac{1}{5}$ hours; hence it is 7 o'clock and 12 minutes A. M.



4. SUG.—In this problem M N represents the time from midnight to noon, M T the time past midnight, and N T the time past noon. Then, $\frac{1}{5}$ of M T = N T, which, subtracted from $\frac{5}{5}$ M T = $\frac{4}{5}$ of M T, which equals M N, or 12 hours; hence $\frac{1}{5}$ of M T = 3 hours, which equals N T; hence it is 3 o'clock, P. M.

11. SUG.—In this problem M′ M represents the time from midnight to midnight, and M′ T, present time. Then, $\frac{3}{5}$ of T M = M′ T, which, added to $\frac{5}{5}$ of T M = $\frac{8}{5}$ of T M, which equals M′ M, or 24 hours; hence $\frac{3}{5}$ of T M, which is M′ T = 9 hours; hence the time is 9 o'clock, A. M.



16. Sug.—At 4 o'clock the hour and minute hands are 4 spaces apart, and, as in problem 14, to gain one space the minute hand goes $\frac{12}{11}$ of a space, to gain 4 4 spaces it must go $4 \times \frac{12}{11} = \frac{48}{11}$, or $4\frac{4}{11}$ spaces; to go one space requires 5 minutes, to go $4\frac{4}{11}$ spaces will require $4\frac{4}{11} \times 5$ minutes $= 21\frac{9}{11}$ minutes.

17. Sug.—We find the time past 9 o'clock $= \frac{2}{3}$ of the time to midnight; then $\frac{3}{3} + \frac{2}{3}$, or $\frac{5}{3}$ of the time to midnight, equals the time from 9 o'clock till midnight, which is 15 hours, &c.

28. Sug.—We find that the time past noon equals $\frac{6}{5}$ of the time to midnight minus $\frac{6}{5}$ of an hour, hence $\frac{11}{5}$ of the time to midnight minus $\frac{6}{5}$ of an hour equals 12 hours, and $\frac{11}{5}$ of the time to midnight will equal $\frac{66}{5}$ hours, hence $\frac{5}{5}$ of the time to midnight equals 6 hours; therefore it is 6 o'clock, P. M.

29. Sug.—We find the two parts to be 40 and 50, then after cutting a piece from the longer, $\frac{1}{5}$ of 40, or 8 feet $= \frac{1}{4}$ of the remainder, hence the remainder is 32 feet; hence there must be 50 — 32, or 18 feet cut from the longer.

LESSON IX.

6. Sol.—If 8 men pay for the coach, each man pays $\frac{1}{8}$ of the price, but when they take in 4 persons there are 12 in all, and each pays $\frac{1}{12}$ of the price; hence $\frac{1}{8}$ of the price minus $\frac{1}{12}$ of the price, which is $\frac{1}{24}$ of the price, equals $\$\frac{3}{4}$, and $\frac{24}{24}$ of the price equals 24 times $\$\frac{3}{4}$, or $18.

6. Sol. 2d.—If the expense of one man is diminished $\$\frac{3}{4}$, the expense of 8 men is diminished 8 times $\$\frac{3}{4}$, or $6; hence 4 men pay $6, and 1 man pays $\frac{1}{4}$ of $6, or $\$\frac{6}{4}$, and 8 + 4, or 12 men, would pay 12 times $\frac{6}{4}$, or $18.

9. Sol.—If 3 times the square of his age is 75 years, the square of his age equals $\frac{1}{3}$ of 75 years, or 25 years, and his age equals the square root of 25 years, which is 5 years.

12. Sol.—The square of twice a number $(2\text{ No.})^2$ equals 4 times the square of the number, which equals 256; hence the square of the number equals $\frac{1}{4}$ of 256, or 64, and the number equals the square root of 64, or 8.

13. Sug.—If the square of $\frac{2}{5}$ of the number equals 100, $\frac{2}{5}$ of the number equals the square root of 100, which is 10, &c.

16. Sug.—The square of $\frac{1}{2}$ the number equals $\frac{1}{4}$ of the square of the number, and $\frac{3}{4}$ of this is $\frac{3}{16}$ of the square of the number, &c.

19. Sug.—The cube of $\frac{2}{3}$ of the number equals $\frac{8}{27}$ of the cube of the number; hence $\frac{2}{3}$, or $\frac{18}{27}$, of the cube of the number, minus $\frac{8}{27}$ of the cube, which is $\frac{10}{27}$ of the cube of the number, equals 10, &c.

21. Sug.—$\frac{2}{3}$ of the number multiplied by $\frac{3}{4}$ of the number equals $\frac{1}{2}$ of the square of the number, and the square of $\frac{1}{2}$ of the number equals $\frac{1}{4}$ of the square of the number, &c.

LESSON X.

The first problem in this lesson, and others of the same class, may be solved by the following method, which will be preferred by some to the method given in the text-book.

1. Sol.—When for the mixture I use 1 lb. worth 6*s.* I will gain 3*s.*, and when I use 1 lb. worth 11*s.* I lose 2*s.*; hence to lose 1*s.* I must use $\frac{1}{2}$ lb., and to lose 3*s.*, what I gained on the other, I must use 3 times $\frac{1}{2}$, or $\frac{3}{2}$ lb.; hence I use 3 lbs. of the first as often as $\frac{3}{2}$ lb. of the second, or, in whole numbers, 2 lbs. of the first as often as 3lbs. of the second.

5. Sug.—By the *first* method I find that I must use, for the mixture, 3 lbs. at 6 cents as often as 2 lbs. at 11 cents; hence as often as I use 4 lbs. at 11 cents, I must use 2 times 3 lbs., or 6 lbs., at 6 cents.

5. Sug.—By the *second* method I find that on 1 lb. at 11 cents, I lose 3 cents, and on 4 lbs. I lose 12 cents; also, on 1 lb. at 6 cents, I gain 2 cents; hence, to gain 12 cents, what I lost on 4 lbs. at 11 cents, I must use as many lbs. as 2 is contained times in 12, or 6 lbs. at 6 cents.

10. Sug.—By comparing I find I must take 2 lbs. at 6 cents as often as 3 lbs. at 11 cents; also, that I must take 1 lb. at 7 cents as often as 1 lb. at 11 cents; hence when I take 2 lbs. at 6 cents, and 1 lb. at 7 cents, I must take $2 + 1$, or 3 lbs., at 11 cents.

Rem.—By the *second* method, if I take 1 lb. at 6 cents and 1 lb. at 7 cents, I will gain $3 + 2$, or 5 cents; if I take 1 lb. at 11 cents I will lose 2 cents; to lose 5 cents, what I have gained, I must take $\frac{5}{2}$ lbs.; hence the proportion is 1, 1, $\frac{5}{2}$, or 2, 2, 5.

17. Sug.—We see he travelled 10 miles more the sixth day, which is 5 days after the first, than on the first day; hence he travelled $\frac{1}{5}$ of 10, or 2 miles more each day, after the first, than on the preceding day.

18. Sug.—He travelled 10 miles more the last day than the first, hence he travelled as many days, after the first, as 2 is contained times in 10, or 5; therefore he travelled 6 days.

LESSON XI.

7. Rem.—Problems of this kind may be solved by two different processes, which we distinguish as the *direct* and the *indirect* methods. In the direct method we commence at the beginning of the problem; in the indi-

rect method we commence at the latter part of the problem, and work toward the beginning. The second, or indirect method, is preferred. We give both.

7. Sol. 1st.—After borrowing as much as he had at first, he had twice his money, and then spending 4 cents, he had twice his money minus 4 cents. At the second store, after borrowing as much as he had, he had 4 times his money at first, minus 8 cents, and after spending 4 cents, he had 4 times his money minus 12 cents, which equals 4 cents, &c.

7. Sol. 2d.—Since he had 4 cents remaining before spending 4 cents, at the second store, he had 4 cents + 4 cents, or 8 cents; but $\frac{1}{2}$ of this he borrowed, hence he had $\frac{1}{2}$ of 8 cents, or 4 cents, when he left the first store; but he had spent 4 cents there, hence, before spending these 4 cents, he had 4 + 4, or 8 cents; but one half of this he had just borrowed, hence, at first, he had $\frac{1}{2}$ of 8 cents, or 4 cents.

13. Sug.—At each store he borrowed 2 cents more than he spent; hence at 3 stores he would borrow 3 times 2 cents, or 6 cents more than he spent, and since this doubled his money, he at first had 6 cents.

LESSON XII.

7. Sol.—Since A receives \$24, and they together receive \$60, B receives \$60 — \$24, or \$36. Had A mowed twice as much as B he would have received twice \$36, or \$72, but he only received \$24, hence for mowing 8 acres he should receive \$72 — \$24, or \$48, and for mowing one acre he should receive $\frac{1}{8}$ of \$48, or \$6; hence A, who received \$24, mowed as many acres as 6 is contained times in 24, which are 4, and B, who received \$36, mowed as many acres as 6 is contained times in 36, which are 6.

12. SUG.—We find $\frac{4}{3}$ of the time to midnight 2 hours hence, + 2 hours + 2 hours, equals 12 hours; hence, $\frac{4}{3}$ of the time to midnight, 2 hours hence, equals 8 hours, and $\frac{1}{3}$ of the time to midnight 2 hours hence, which was the time past noon 2 hours ago, equals $\frac{1}{4}$ of 8 hours, or 2 hours; hence the time now is 2 hours plus 2 hours, or 4 hours past noon, or 4 o'clock, P. M.

13. SUG.—We find F's age *now*, + 1 year, equals E's age 4 years ago, and F's age *now*, + 4 years, equals F's age 4 years hence; therefore F's age now, + 4 years = $\frac{1}{2}$ of (F's now + 4 years), or $\frac{1}{2}$ of F's age now, + 2 years; hence F's now minus $\frac{1}{2}$ of F's now, which is $\frac{1}{2}$ of F's age now, equals 4 — 2, or 2 years, &c.

17. SUG.—After 7 jumped from the first field into the second, there were 14 more in the second than in the first; hence 3 times the number in the first field, minus once the number in the first field, which is twice the number in the first field, equals 14, and the number in the first field is 7; hence 7 + 7, or 14, was the number in each field after the $\frac{1}{3}$ were sold; therefore 14 = $\frac{2}{3}$ of the number in each field at first.

20. SOL.—Since B received $30, and both received $50, A received $50 — $30, or $20. Since $\frac{4}{5}$ of what A digs, + 4 rods equals $\frac{2}{3}$ of what B digs, $\frac{4}{5}$ of what A receives, + the cost of digging 4 rods, equals $\frac{2}{3}$ of what B receives, that is—$\frac{4}{5}$ of $20, + the cost of digging 4 rods equals $\frac{2}{3}$ of $30, or $16, + the cost of digging 4 rods equals $20, hence the cost of digging 4 rods is $20 — $16, or $4, and the cost of digging 1 rod is $1; therefore A, who received $20, dug as many rods as 1 is contained times in 20, &c.

LESSON XIII.

REM.—The problems in this lesson, which are similar to the first, may also be solved by another method, which some may prefer to the one given in the Arithmetic. The author prefers the one given, however, for several reasons. It seems more analytic, less algebraic, and does not involve fractional parts of the numbers given, as the other method does in many of the problems.

1. SOL. 2d.—After losing 15 cents he had his money minus 15 cents, and after finding $\frac{1}{3}$ as much as he lost, he had $\frac{4}{3}$ of his money, minus $\frac{4}{3}$ of 15 cents, which, by the condition of the problem, equals $\frac{1}{2}$ of his money; hence $\frac{4}{3}$ of his money, minus $\frac{1}{2}$ of his money, which is $\frac{5}{6}$ of his money, equals 15 cents, &c.

13. SUG.—If 3 persons eat 5 loaves, each eats $\frac{5}{3}$ of a loaf; hence A furnished C with $2 - \frac{5}{3}$, or $\frac{1}{3}$ of a loaf, and B furnished C with $3 - \frac{5}{3}$, or $\frac{4}{3}$ of a loaf. If for $\frac{5}{3}$ of a loaf C pays 20 cents, for $\frac{1}{3}$ of a loaf, what A furnishes, he will pay $\frac{1}{5}$ of 20 cents, or 4 cents, &c.

REM.—This problem is sometimes solved by dividing the money that C pays between A and B, in proportion to the number of loaves each furnishes, instead of dividing in the proportion of the number of loaves each furnishes C. Thus they would divide the 20 cents between A and B, in the proportion of 2 to 3, the number of loaves which A and B furnish respectively.

17. SUG.—We find that A and B each eat 4 eggs, and C eats 8 eggs; hence A furnishes C with $6 - 4$, or 2 eggs, and B furnishes C with $10 - 4$, or 6 eggs. If C pays 16 cents for 8 eggs, for one egg he will pay 2 cents, and for 2 eggs, the number that A furnishes, he will pay 4 cents, and for 6 eggs, the number that B furnishes, he will pay 12 cents.

LESSON XIV.

7. Sol.—If for $\frac{4}{5}$ of the remainder, minus 8, he received \$4, for $\frac{1}{5}$ of the remainder, minus 2, he received \$1, and for $\frac{5}{5}$ of the remainder, minus 10, he would receive \$5; hence 12 + 10, or 22, are worth \$16 — \$5, or \$11, one is worth \$$\frac{1}{2}$, and for \$16 he could buy as many as $\frac{1}{2}$ is contained times in 16, which are 32.

11. Sol.—If he sells $\frac{1}{2}$ of the first remainder, minus 8, for \$22, he would receive for $\frac{2}{2}$ of the remainder, minus 8, twice \$22, or \$44; hence 8 are worth \$60 — \$44, or \$16, one is worth \$2, and for \$60 he could buy 30, and since the dog had killed $\frac{1}{4}$ of the sheep, 30 is $\frac{3}{4}$ of the number he had at first, &c.

12. Sug.—We find if he sold the remainder for cost he would receive 60 dimes, and if he kept 15, and sold the remainder for cost, he would receive 30 dimes; hence the 15 are worth 60 — 30, or 30 dimes, and for 60 dimes he could buy 30; hence 30 = $\frac{3}{5}$ of his number at first.

14. Sug.—We find the remainder + 10 are worth 80 dimes, and the remainder — 10 are worth 40 dimes; hence 10 + 10, or 20, cost 80 — 40, or 40 dimes, one cost 2 dimes, 80 dimes will buy 40; hence 40 — 10, or 30, equals $\frac{1}{4}$ the number at first.

15. Sol.—If 10 lbs. of the mixture contain $\frac{1}{5}$ of a pound of salt, to contain $\frac{5}{5}$ of a pound of salt will require 5 times 10, or 50 lbs. of the mixture, and to contain 2 lbs. of salt will require 2 times 50 lbs., or 100 lbs. of the mixture; hence there were added 100 — 60 lbs., or 40 lbs. of water.

16. Sol.—We find that for 3 cows there is $\frac{3}{4}$ of an acre of ploughed land, hence $\frac{3}{4}$ of the number of acres pastured equals the number of acres ploughed; hence $\frac{7}{4}$ of the number pastured equals 140, $\frac{4}{4}$ the number pastured = 80, hence there were 80 times 3 cows, or 240 cows.

LESSON XV.

The problems in this lesson, similar to the first, are perhaps rather too difficult to be solved mentally, at the outset; the pupils may, therefore, solve them upon the black-board for a few times, if desirable, before being required to give a mental solution.

5. SUG.—We find that $\frac{8}{5}$ of the daughter's share equals a certain amount, and $\frac{5}{5}$ of her share equals $\frac{5}{8}$ of this amount; also, that $\frac{6}{5}$ of the son's share equals 2 times this amount, and $\frac{5}{5}$ of the son's share equals $\frac{5}{3}$ of this amount; hence the shares are as $\frac{5}{8}$ to $\frac{5}{3}$, or as $\frac{15}{24}$ to $\frac{40}{24}$, or as 15 to 40; hence the daughter's share is \$1500, and the son's share is \$4000.

7. SOL.—After spending $\frac{1}{2}$ of his money + \$$\frac{1}{2}$, there remained $\frac{1}{2}$ of his money — \$$\frac{1}{2}$; then after spending $\frac{1}{2}$ of this plus \$$\frac{1}{2}$, there remained $\frac{1}{2}$ of ($\frac{1}{2}$ his money — \$$\frac{1}{2}$) — \$$\frac{1}{2}$, which is $\frac{1}{4}$ of his money — \$$\frac{1}{4}$ — \$$\frac{1}{2}$, which equals \$3; hence $\frac{1}{4}$ of his money equals \$$3\frac{3}{4}$, and $\frac{4}{4}$ of his money equals \$15.

REM.—Some solve such problems by finding what was given away each time, and then subtract this from what he had to find the remainder. Thus, suppose $\frac{2}{3}$ was given away, they would find $\frac{2}{3}$ of what they had, and subtract this from what they have. The better way is to reason thus: if they gave $\frac{2}{3}$ of what they have away, there would remain $\frac{1}{3}$ of what they have, &c. This is just one-half as long as the other method, and is equally clear.

13. SUG.—A's money was on interest 6 years, hence $\frac{8}{5}$ of A's principal equals his amount, and B's money was on interest 2 years, hence $\frac{6}{5}$ of B's principal equals his amount; but $\frac{2}{3}$ of A's amount equals $\frac{2}{5}$ of B's amount, hence A's amount equals $\frac{3}{5}$ of B's amount, hence $\frac{8}{5}$ of A's principal equals $\frac{3}{5}$ of B's amount, and $\frac{5}{5}$ of A's principal equals $\frac{3}{8}$ of B's amount, and since $\frac{6}{5}$ of B's principal

equals B's amount, $\frac{5}{5}$ of B's principal equals $\frac{5}{6}$ of B's amount, hence A's principal is to B's as $\frac{3}{8}$ to $\frac{5}{6}$, or as $\frac{9}{24}$ to $\frac{20}{24}$, or as 9 to 20; hence A's share is \$90 and B's share is \$200.

14. Sol.—By a condition of the problem 7 times the value of the silver watch equals the value of the gold watch and chain, but the gold watch equals 3 times the value of the silver watch and chain, therefore 7 times the value of the silver watch equals 3 times the value of the silver watch, plus 4 times the value of the chain; hence 7 times — 3 times, or 4 times the value of the silver watch equals 4 times the value of the chain, therefore once the value of the silver watch equals the value of the chain, or \$20, &c.

LESSON XVI.

23. Sol.—He bought the goods for 100 per cent., minus 20 per cent., which is 80 per cent. of par value, and sold them for 100 + 20, or 120 per cent. of par value; hence he gained 120 — 80, or 40 per cent. on 80 per cent., and therefore he gained $\frac{1}{2}$ of the cost, which equals \$90, and $\frac{2}{2}$ of the cost equals twice \$90, or \$180.

Rem.—This problem is frequently solved incorrectly, by regarding \$90 as 40 per cent., or $\frac{2}{5}$ of the money invested, from which the money invested is found to be \$225.

25. Sug.—We find he sold it for $\frac{1}{3}$ of 120 per cent., or 40 per cent.; hence he lost 100 — 40, or 60 per cent.

29. Sug.—I sell the hay for \$12 a ton; hence \$12 was $\frac{4}{5}$ of the price asked, and the price asked was therefore \$15 a ton.

33. Sug.—I retail for 150 per cent., and sell at wholesale for $\frac{1}{4}$ less, or $\frac{3}{4}$ of 150 per cent., or $112\frac{1}{2}$ per cent.; hence I gain $12\frac{1}{2}$ per cent.

38. SUG.—There remained 80 per cent., and this was sold for $\frac{7}{5}$ of 80 per cent. of the cost of the whole, which is 112 per cent.; hence the gain was 12 per cent.

45. SUG.—Since the difference between the fortunes is once C's fortune, the difference between the amounts is once C's amount, which is $\frac{3}{2}$ of C's principal; hence $\frac{3}{2}$ of C's fortune equals $330, &c.

45. Another method, which is more general, is as follows: We find that $\frac{3}{2}$ of C's principal equals C's amount, and $\frac{6}{2}$ of C's principal equals B's amount; then $\frac{6}{2}$ of C's principal, minus $\frac{3}{2}$ of C's principal, which is $\frac{3}{2}$ of C's principal, equals $330, &c.

LESSON XVII.

8. SUG.—After working 9 days, $\frac{3}{5}$ of the time required, A and B could complete it in 6 days, but A, B, and C completed it in 4 days; hence C could do $\frac{1}{4} - \frac{1}{6}$, or $\frac{1}{12}$ of the remainder in one day, and he will do the remainder in 12 days; and if he could do $\frac{2}{5}$ of the work in 12 days, he could do the whole of it in 30 days.

14. REM.—This problem means how much did C gain on D when C had run 40 rods.

24. SUG.—There remained 6 melons, and they were sold for $\frac{4}{3}$ of 36 cents, or 48 cents; hence they were sold for 8 cents apiece.

27. SUG.—He must sell 80 per cent. for 120 per cent.; hence on 80 per cent. he gains 40 per cent., or the gain is 50 per cent.

32. SUG.—$\frac{5}{4}$ of the cost of the first horse equals what was received for it, and $\frac{3}{4}$ of the cost of the second horse equals what was received for it; and since the second horse cost $\frac{2}{3}$ as much as the first, $\frac{3}{4}$ of the cost of the

second horse equals $\frac{2}{4}$ of the cost of the first, which, added to $\frac{5}{4}$ of the cost of the first, equals $\frac{7}{4}$ of the cost of the first, which equals $210, &c.

33. Sug.—We find that $\frac{4}{5}$ of the cost of the horse equals what it was sold for, and $\frac{5}{4}$ of the cost of the carriage equals what it was sold for, but $\frac{4}{5}$ of the cost of the horse equals $\frac{2}{3}$ of the cost of the carriage; then $\frac{2}{3}$ of the cost of the carriage, plus $\frac{5}{4}$ of the cost of the carriage, which is $\frac{23}{12}$ of the cost of the carriage, equals $230, &c.

CONTRACTIONS

AND

ABBREVIATED PROCESSES.

The improved methods of teaching Arithmetic at the present day, cultivate the reasoning faculties, but neglect, to some extent, to give that readiness in mechanical computation that was secured by the old methods. Pupils seem to acquire a distaste for the mechanical operations of Addition, Multiplication, &c., and teachers are somewhat to blame for it.

Arithmetic consists of a reasoning and a mechanical part, and both should be thoroughly taught, since both are necessary to make the accomplished arithmetician. The utility of readiness in the mechanical operations is felt in the extraction of roots, the computation of logarithms, and logarithmic sines, &c., and in the calculation of eclipses and the occultation of stars. We propose to suggest a number of exercises for the use of pupils in acquiring readiness in the mechanical processes.

ADDITION AND SUBTRACTION.

Quite a large number of methods is suggested in the *Normal Primary Arithmetic.* We here give a few more, both for Mental and Written Arithmetic.

1. Require pupils to add mentally such problems as the following:

$25 + 37$; $82 + 69$; $76 + 57$; $124 + 367$; $256 + 385$; $467 + 583$; $73 + 85 + 97$; $125 + 327 + 562$.

REMARK.—With a little practice pupils will perform such problems with great facility and accuracy, and such ability will be of much advantage to them in the practical affairs of life. Let many such problems be given by the teacher, increasing in difficulty with the advancement of the class.

2. The following exercises will also be valuable. Find the value of

$5 \times 4 + 6 \times 7$; $6 \times 9 + 7 \times 8$; $3 \times 9 + 4 \times 7$; $4 \times 6 + 3 \times 9 + 8 \times 5$; $8 \times 3 + 9 \times 6 + 5 \times 9$.

REMARK.—The teacher may exercise his pupils for quite a long time on problems of this character, making them more difficult as the class becomes prepared for them. Such exercises are important, also, since they prepare the student for an abbreviated method of Multiplication.

3. The teacher may arrange columns of figures upon the board, as is suggested in the "Normal Primary Arithmetic," page 31, and assign with these columns problems similar to the above.

4. In Written Arithmetic require the pupils to add, not only one column at a time, but *two*, and even *three*, at the same time. With practice, *two* columns may be added at the same time very easily. Many skilful accountants run up two columns of their books with much ease.

Many of the exercises which we have suggested for addition can also be used in subtraction. Let exercises in addition and subtraction be combined.

The above exercises, and others that the ingenious teacher may devise, will be found very valuable in checking the tendency of our improved methods of instruction, to ignore the utility of the mechanical processes of Arithmetic.

MULTIPLICATION.

The following exercises, in Multiplication, are presented for the purpose of acquiring readiness in the mechanical process of multiplying. Some of the following problems may be solved entirely mentally, others are adapted to slate and black-board exercises.

1. To multiply when the multiplicand and multiplier consists of several figures.

1. What is the product of 46×5? 289×6? 572×8? 896×7? 785×5? 469×9? 25×34? 87×53? 98×37? 123×234?

Many problems in Multiplication may be solved by deriving a law by which the product arises. The following are presented as an illustration.

2. To multiply two numbers between 10 and 20.

METHOD.—*Arrange, from right to left, the right hand figure of the product of the units, the left hand figure of this product, plus the sum of the units, and then the product of the tens.*

DEM.—This law can easily be derived by multiplying together two numbers, as 16 by 18; thus, $16 \times 18 = \overset{h}{1} - (8 + \overset{t}{6} + 4) - \overset{u}{8} = 288$.

What is the value of

12×13? 15×14? 16×17? 13×18? 17×19? 15×18?

3. To multiply two numbers of two places when the unit figure is 1 in each.

METHOD.—*Arrange, from right to left, the unit figure, the sum of the tens, and the product of the tens.*

DEM.—This law can be readily derived by multiplying together two numbers, as 41 and 71; thus, $41 \times 71 = \overset{h}{7 \times 4} - \overset{t}{(7 + 4)} - \overset{u}{1} = 2911.$

What is the value of

21×31? 21×51? 51×41? 71×81? 91×21? 81×91?

4. To multiply two numbers of two places each, in which the unit figures are alike, and the sum of the tens is 10.

METHOD.— *Write from right to left the product of the units, and the product of the tens increased by the unit figure of the numbers.*

DEM.—This can be readily shown by multiplying together two numbers, as 36 and 76; thus, $76 \times 36 = \overset{h}{(7 \times 3)}, - \overset{t}{(3 \times 6 + 7 \times 6)}, - \overset{u}{(6 \times 6)} = \overset{h}{(7 \times 3)}, - \overset{t}{(10 \times 6)}, - \overset{u}{6 \times 6} = 2736.$

What is the value of

42×62? 37×77? 85×25? 98×18? 79×39? 39×79?

5. To multiply two numbers of two places, when the sum of the units is 10, and the difference of the tens is 1.

METHOD.— *Write, from right to left,* 100, *minus the square of the units of the larger number, and to the right of this the square of the tens of the larger number diminished by* 1.

DEM.—This may be readily shown by multiplying together two numbers, as 57 and 63, thus:

$$57 \times 63 = (6)^2 - 1 - (100 - (3)^2) = 3591.$$

What is the value of

46×54? 37×43? 52×68? 79×61? 86×94? 71×89?

6. To multiply any integer $+ \frac{1}{2}$ by itself.

METHOD.—*Multiply the whole number by the next larger whole number, and to the product add $\frac{1}{4}$.*

DEM.—This may be proved by multiplying any two such mixed numbers together, as $7\frac{1}{2}$ and $7\frac{1}{2}$.
$7\frac{1}{2} \times 7\frac{1}{2} = (7 + \frac{1}{2})(7 + \frac{1}{2}) = (7 \times 7 + (\frac{1}{2} \times 7 + \frac{1}{2} \times 7) + \frac{1}{2} \times \frac{1}{2} = 7 \times 7 + 1 \times 7 + \frac{1}{4} = (7 \times 1)\, 7 + \frac{1}{4} = 8 \times 7 + \frac{1}{4}$, which equals $56\frac{1}{4}$.

What is the value of

$8\frac{1}{2} \times 8\frac{1}{2}$? $9\frac{1}{2} \times 9\frac{1}{2}$? $11\frac{1}{2} \times 11\frac{1}{2}$? $14\frac{1}{2} \times 14\frac{1}{2}$? $15\frac{1}{2} \times 15\frac{1}{2}$? $17\frac{1}{2} \times 17\frac{1}{2}$? $25\frac{1}{2} \times 25\frac{1}{2}$? $75 \times 75 = 7\frac{1}{2}$ tens $\times$ $7\frac{1}{2}$ tens? 95×95?

7. To multiply two mixed numbers when the integers are the same, and the sum of the fractions equals 1.

METHOD.—*Multiply the whole number by the next larger whole number, and to the product add the product of the fractions.*

DEM.—This may be readily derived by multiplying together two such numbers as $5\frac{3}{7}$ and $5\frac{4}{7}$; thus, $5\frac{3}{7} \times 5\frac{4}{7} = (5 + \frac{3}{7})(5 + \frac{4}{7}) = 5 \times 5 + (\frac{3}{7} \times 5 + \frac{4}{7} \times 5) + \frac{3}{7} \times \frac{4}{7} = 5 \times 5 + 1 \times 5 + \frac{3}{7} \times \frac{4}{7} = 6 \times 5 + \frac{12}{49} = 30\frac{12}{49}$.

What is the value of

$6\frac{2}{5} \times 6\frac{3}{5}$? $9\frac{5}{8} \times 9\frac{3}{8}$? $15\frac{7}{11} \times 15\frac{4}{11}$? $20\frac{5}{12} \times 20\frac{7}{12}$? $25\frac{3}{10} \times 25\frac{7}{10}$? $30\frac{5}{13} \times 30\frac{8}{13}$? $40\frac{7}{15} \times 40\frac{8}{15}$? $50\frac{1}{50} \times 50\frac{49}{50}$?

8. To multiply two mixed numbers when the difference of the integers is a unit, and the sum of the fractions is a unit.

METHOD.—*First square the larger number, and diminish this by 1; secondly, square the fraction of the larger number, subtract this from 1, and unite the result with the former result.*

DEM.—This law may be derived by a process similar to those already explained.

What is the value of

$3\frac{1}{2} \times 4\frac{1}{2}$? $5\frac{1}{3} \times 6\frac{2}{3}$? $7\frac{1}{4} \times 6\frac{3}{4}$? $8\frac{2}{5} \times 9\frac{3}{5}$? $9\frac{5}{6} \times 10\frac{1}{6}$?

$12\frac{1}{9} \times 13\frac{8}{9}$? $15\frac{6}{7} \times 16\frac{1}{7}$? $17\frac{2}{9} \times 18\frac{7}{9}$? $19\frac{4}{5} \times 18\frac{1}{5}$? $\times 20\frac{1}{3} \times 21\frac{2}{3}$?

9. To find the product of any two mixed numbers, whose fractional parts are halves.

METHOD.—*Take the product of the integers, increase this by $\frac{1}{2}$ of their sum, and by $\frac{1}{4}$.*

DEM.—This law can be readily obtained by a process similar to those already given.

What is the value of

$3\frac{1}{2} \times 5\frac{1}{2}$? $6\frac{1}{2} \times 2\frac{1}{2}$? $7\frac{1}{2} \times 5\frac{1}{2}$? $6\frac{1}{2} \times 9\frac{1}{2}$? $10\frac{1}{2} \times 5\frac{1}{2}$? $8\frac{1}{2} \times 12\frac{1}{2}$? $7\frac{1}{2} \times 11\frac{1}{2}$? $5\frac{1}{2} \times 15\frac{1}{2}$? $18\frac{1}{2} \times 16\frac{1}{2}$? $20\frac{1}{2} \times 30\frac{1}{2}$?

10. To find the square of a mixed number, whose fractional part is $\frac{1}{4}$.

METHOD.—*First, when the integer is* EVEN, *square the integer, add $\frac{1}{2}$ of itself and the square of $\frac{1}{4}$.*

Secondly, when the integer is ODD, *square the whole number, add $\frac{1}{2}$ of the next smaller number, and also $\frac{9}{16}$.*

DEM.—This law may be derived by a process similar to those already given.

What is the square of

$4\frac{1}{2}$? $6\frac{1}{2}$? $8\frac{1}{2}$? $10\frac{1}{2}$? $12\frac{1}{2}$? $14\frac{1}{2}$? $16\frac{1}{2}$? $18\frac{1}{2}$? $20\frac{1}{2}$? $24\frac{1}{2}$? $5\frac{1}{2}$? $7\frac{1}{2}$? $9\frac{1}{2}$? $11\frac{1}{2}$? $13\frac{1}{2}$? $15\frac{1}{2}$? $17\frac{1}{2}$? $19\frac{1}{2}$? $21\frac{1}{2}$? $25\frac{1}{2}$?

11. To find the square of a mixed number, whose fractional part is $\frac{3}{4}$.

METHOD.—*First, when the number is* EVEN, *square the whole number, increase this by $\frac{3}{2}$ of the whole number, and by the square of $\frac{3}{4}$.*

Secondly, when the number is ODD, *square the whole number, increase this by $\frac{3}{2}$ of the next smaller number, also by 2 and by $\frac{1}{16}$.*

What is the square of

$4\frac{3}{4}$? $6\frac{3}{4}$? $8\frac{3}{4}$? $10\frac{3}{4}$? $12\frac{3}{4}$? $14\frac{3}{4}$? $18\frac{3}{4}$? $20\frac{3}{4}$? $24\frac{3}{4}$? $30\frac{3}{4}$? $7\frac{3}{4}$? $9\frac{3}{4}$? $11\frac{3}{4}$? $15\frac{3}{4}$? $17\frac{3}{4}$? $19\frac{3}{4}$? $25\frac{3}{4}$? $27\frac{3}{4}$? $33\frac{3}{4}$? $41\frac{3}{4}$?

12. To square any number whose unit figure is 5.

METHOD.—*Multiply the part preceding the units by itself, increased by a unit, and prefix the product to* 25; *thus, in squaring* 65, *we have* 6 × 7, *or* 42, *which, prefixed to* 25, *gives* 4225.

DEM.—This law may be also derived by a process similar to those already given.

What is the square of

25? 35? 45? 55? 75? 85? 95? 105? 115? 125? 135? 145? 155? 165? 175? 185? 195? 205? 225?

13. To find the product of any two numbers whose unit figures are 5.

METHOD.—*Take the product of the figures preceding the* 5 *in each number, increase this by* $\frac{1}{2}$ *of the sum of these figures, and prefix the result to* 25.

DEM.—This method is also derived by a process similar to those already explained.

REMARK.—If the sum of the figures preceding the 5 is ODD, when we take $\frac{1}{2}$ of it, the $\frac{1}{2}$, or 5 tens, which remains, must be added to the figure 2 of the 25, or we may take $\frac{1}{2}$ of the next smaller number, and use 75 as the suffix.

What is the value of

25 × 45? 55 × 75? 75 × 95? 65 × 95? 35 × 85? 85 × 45? 155 × 35? 165 × 45? 185 × 65? 175 × 65? 225 × 105?

14. To find the square of any number ending in 25.

METHOD.—*Regard the* 25 *as* $\frac{1}{4}$, *and then proceed as in the case of squaring a mixed number, whose fractional part is* $\frac{1}{4}$, *and in the result reduce the number of sixteenths, which is* $\frac{1}{16}$ *of a number of thousands, to tens and units.*

What is the square of

225? 425? 625? 825? 1025? 1225? 2025? 125? 325? 525? 725? 925? 1325? 2125?

15. To multiply any numbers in which the sum of the units and tens equals 100.

METHOD.—*This is similar to multiplying two mixed numbers, in which the sum of the fractions equals a unit. We regard the number expressed by the units and tens as so many hundredths, and proceed as in a previous case.*

REMARK.—This method is particularly adapted to two numbers, one of which ends in 25, and the other in 75, since these are respectively equal to $\frac{1}{4}$ and $\frac{3}{4}$.

What is the value of

125×175? 225×475? 675×825? 975×725? 1025×1275? 125×275? 375×425? 325×875? 825×575? 1375×1425?

16. To find the square of any number ending in 75.

METHOD.—*We regard the* 75 *as* $\frac{3}{4}$, *and then proceed as in the case of squaring a mixed number, whose fractional part is* $\frac{3}{4}$.

17. To multiply by aliquot parts of 10, 100, 1000, &c., as $2\frac{1}{2}$; $3\frac{1}{3}$; 5; $12\frac{1}{2}$; $16\frac{2}{3}$; 25; $33\frac{1}{3}$; 50; 125; $166\frac{2}{3}$; 250; $333\frac{1}{3}$; $833\frac{1}{3}$.

18. To multiply any number by $2\frac{1}{2}$.

METHOD.—*We annex a cipher to the right of the multiplicand, and divide by* 4.

DEM.—$2\frac{1}{2} = \frac{10}{4}$, hence $2\frac{1}{4}$ times a number equals $\frac{1}{4}$ of 10 times the number.

What is the value of

$4 \times 2\frac{1}{2}$? $16 \times 2\frac{1}{2}$? $28 \times 2\frac{1}{2}$? $156 \times 2\frac{1}{2}$? $258 \times 2\frac{1}{2}$? $364 \times 2\frac{1}{2}$?

19. To multiply any number by $3\frac{1}{3}$.

METHOD.—*We annex a cipher and divide by* 3.

DEM.—This and the following methods may be demonstrated in a manner similar to the preceding.

What is the value of
$9 \times 3\frac{1}{3}$? $27 \times 3\frac{1}{3}$? $837 \times 3\frac{1}{3}$? $757 \times 3\frac{1}{3}$? $586 \times 3\frac{1}{3}$?

20. To multiply by 5.

METHOD.—*Annex one cipher and divide by* 2.

REMARK.—The teacher can form problems for himself, illustrating the other cases.

21. To multiply by $12\frac{1}{2}$.

METHOD.—$12\frac{1}{2}$ *being* $\frac{1}{8}$ *of* 100, *we annex two naugnts* and divide by 8.

22. To multiply by $16\frac{2}{3}$.

METHOD.—$16\frac{2}{3}$ *being* $\frac{1}{6}$ *of* 100, *we annex two naughts and divide by* 6.

23. To multiply by 25.

METHOD.—25 *being* $\frac{1}{4}$ *of* 100, *we annex two naughts and divide by* 4.

24. To multiply by $33\frac{1}{3}$.

METHOD.—$33\frac{1}{3}$ *being* $\frac{1}{3}$ *of* 100, *we annex two naughts and divide by* 3.

25. To multiply by 50.

METHOD.—*Since* 50 *is* $\frac{1}{2}$ *of* 100, *we annex two naughts and divide by* 2.

26. To multiply by 125, $166\frac{2}{3}$, 250, $333\frac{1}{3}$.

METHOD.—*Since these numbers are respectively* $\frac{1}{8}$, $\frac{1}{6}$, $\frac{1}{4}$, *and* $\frac{1}{3}$ *of* 1000, *we annex three naughts, and divide respectively by* 8, 6, 4, *and* 3.

REMARK.—Other methods, somewhat similar to these, may be obtained by the teacher; we give one as an illustration.

27. To multiply by $133\frac{1}{3}$.

METHOD.—*Since* $133\frac{1}{3}$ *is* $\frac{4}{3}$ *of* 100, *we annex two* *naughts to the multiplicand, divide by 3, and add the quotient to the dividend; thus,* 369 × 133⅓ = 36900 ÷ 3 + 36900 = 49200.

The work may be expressed thus:—

$$\begin{array}{r} 3)36900 \\ \hline 12300 \\ \hline 49200 \textit{ Ans.} \end{array}$$

What is the value of

[illegible] × 133⅓? 684 × 133⅓? 828 × 133⅓? 576 × 133⅓?

28. To multiply by a number consisting of nines.

METHOD.—*Annex as many naughts to the multiplicand as there are 9's in the multiplier, and subtract the multiplicand from the product.*

DEM.—9 times a number = (10 — 1) times the number = 10 times the number minus once the number; in the same way, 99 times a number equals 100 times the number, minus once the number, &c.

What is the value of

28 × 9? 215 × 99? 316 × 99? 282 × 99? 351 × 999? 462 × 99? 715 × 999? 1867 × 9999? 7857 × 9999?

29. To multiply by any number in which all the digits are the same.

METHOD.—*Multiply by a number consisting of as many 9's as there are similar digits, and take such a part of the product as the digit of the multiplier is of* 9.

DEM.—Suppose we wish to multiply by some such number as 444; this number is $\frac{4}{9}$ of 999; hence $\frac{4}{9}$ of 999 times a number, equals 444 times the number.

REMARK.—This is convenient when the similar digits are 1, 3, or 6, in which case $\frac{1}{9}$, $\frac{1}{3}$, and $\frac{2}{3}$ are the respective parts of the product of 9's taken.

30. In the ordinary processes of multiplication, we

obtain partial products, and then add these together for the entire product. With a little practice, however, we may multiply by a number consisting of several figures without writing the partial products. There have been those who could multiply by a number consisting of 10 or 12 digits, writing the result under the given numbers with great readiness. This is a very unusual degree of proficiency; but almost any one can learn to do the same with a multiplier consisting of from 2 to 6 places. We indicate the method by the following problem and its solution.

1. Multiply 5642 by 345.

$$\begin{array}{r} 5642 \\ 345 \\ \hline 1946490 \end{array}$$

EXPLANATION.—1st. $5 \times 2 = 10$, we write 0, and reserve the 1 to "*carry.*"

2d. $5 \times 4 + 4 \times 2 + 1 = 29$, write 9, and reserve the 2 to "*carry.*"

3d. $5 \times 6 + 4 \times 4 + 3 \times 2 + 2 = 54$, write 4, reserve the 5 to carry.

4th. $5 \times 5 + 4 \times 6 + 3 \times 4 + 5 = 66$, write 6, reserve the 6 to carry.

5th. $4 \times 5 + 3 \times 6 + 6 = 44$, write 4, reserve 4 to carry.

6th. $3 \times 5 + 4 = 19$; write this, since it is the last product.

The above work, of course, is performed mentally; the products, and the sum of the products, obtained as we have indicated. With practice, a person may multiply in this manner with much rapidity and accuracy.

DIVISION.

Many of these processes, which we have given for multiplication, may, by a little change, be applied to division. In dividing the aliquot parts of 10, 100, &c., we just reverse the method used in multiplication.

To divide by $2\frac{1}{2}$, we multiply by 4 and divide by 10.

To divide by $16\frac{2}{3}$, we multiply by 6 and divide by 100.

To divide by 25, we multiply by 4 and divide by 100.

In a similar manner, we divide by the other aliquot parts of 10, 100, &c.

Other methods may be presented by the teacher; and we suggest that the exercise will be valuable to the student.

SOCIAL ARITHMETIC.

UNDER the head of Social Arithmetic, we give a collection of problems, particularly adapted to a social circle, or the fireside of a winter evening. The most of these problems are in the form of puzzles, and some of them particularly amusing. The majority of them are very old, their parentage being entirely unknown; so that no credit can be given to their authors. Quite a number of them, however, have not previously been published.

1. Think of a number of 3 or more figures, divide by 9, and name the remainder; erase one figure of the number, divide by 9, and tell me the remainder, and I will tell you what figure you erased.

METHOD.—If the second remainder is less than the first, the figure erased is the difference between the remainders; but if the second remainder is greater than the first, the figure erased equals 9, minus the difference of the remainders.

2. Think of a number, multiply it by 3, and multiply it also by 4, take the sum of the squares of the products, extract the square root of this sum, divide by the first number, and I will name the quotient.

METHOD.—The quotient will always be 5. The same will be also true if we have them multiply and divide by the same multiples of 3, 4, and 5, as 6, 8, 10, &c. If we have them divide by 5, it will give the number they commenced with.

3. Think of a number, multiply it by 5, also by 12; square each product, take their sum, extract the square root, divide by the number commenced with, and I will name the quotient.

METHOD.—The quotient is always 13. To give variety it is well to use multiples of 5, 12; as 10, 24, &c., and then the quotient is 26, &c.

4. Think of a number composed of two unequal digits, invert the digits, take the difference between this and the original number, name one of the digits and I will name the other.

METHOD.—The sum of the digits in the difference is always 9; hence when one is named, the other equals 9 minus the one named.

5. Take any number consisting of three consecutive digits and permutate them, making 6 numbers, and take the sum of these numbers, divide by 6, and tell me the result, and I will tell you the digits of the number taken.

METHOD.—The quotient consists of three equal digits; the digits of the number taken are, 1st, one of these equal digits; 2d, this digit increased by a unit; 3d, this digit diminished by a unit. The same principle holds when the digits of the number taken differ by 2, 3, or 4. It is a very pretty problem to prove that the sum is always divisible by 9, and 18.

6. Think of a number greater than 3, multiply it by 3; if *even*, divide it by 2; if *odd*, add 1, and then divide by 2. Multiply the quotient by 3; if *even*, divide by 2; if *odd*, add 1, and then divide by 2. Now divide by 9 and tell the quotient, without the remainder, and I will tell you the number thought of.

METHOD.—If *even* both times, multiply the quotient by 4; if *even* 2d, and *odd* 1st, multiply by 4, and add 1; if even 1st, and odd 2d, multiply by 4, and add 2; if odd both times, multiply by 4, and add 3.

6. Take any number, divide it by 9, and name the remainder. Multiply the number by some number which I name, and divide this product by 9, and I will name the remainder.

METHOD.—To tell the remainder, I multiply the first remainder by the number by which I told them to multiply the given number, and divide this product by 9. The remainder is the second number that they obtained.

7. A and B have an 8 gallon cask full of wine, which they wish to divide into two equal parts, and the only measures they have are a 5 gallon cask and a 3 gallon cask. How shall they make the division with these two vessels?

8. Two men have 24 ounces of fluid, which they wish to divide between them equally. How shall they effect the division, provided they have only three vessels; one containing 5 oz., the other 11 oz., and the third 13 oz.?

9. Two men, stopping at an oyster saloon, laid a wager as to which could eat the most oysters. One eat ninety-nine, and the other eat a hundred and won. How many did both eat?

REMARK.—The "catch" is in "a hundred and *won*." When this is repeated it sounds as if it meant "one eat 99 and the other eat 101; hence the result usually given is 200. The correct result, of course, is 199.

10. Six ears of corn are in a hollow stump. How long will it take a squirrel to carry them all out, if he takes out three ears a day?

REMARK.—The "catch" is in the word *ears*. He carries out two ears on his head, and one ear of corn each day; hence it will take him 6 days.

11. A and B went to market with 30 pigs each. A sold his at 2 for $1, and B at the rate of 3 for $1, and they, together, received $25. The next day A went to market alone with 60 pigs, and, wishing to sell at the same rate, sold them at 5 for $2, and received only $24. Why should he not receive as much as when B owned half of the pigs?

12. In the bottom of a well, 45 feet deep, there was a frog which commenced travelling towards the top. In his journey he ascended 3 feet every day, but fell back 2 feet every night. In how many days did he get out of the well?

13. A man having a fox, a goose, and some corn,

came to a river which it was necessary to cross. He could, however, take only *one* across at a time, and if he left the goose and corn, while he took the fox over, the goose would eat the corn; but if he left the fox and goose, the fox would kill the goose. How shall he get them all safely over?

14. A man went to a store and purchased a pair of boots worth $5, and hands out a $50 bill to pay for them; the merchant, not being able to make the change, passes over the street to a broker and gets the bill changed, and then returns and gives the man, who bought the boots, his change. After the purchaser of the boots has been gone a few hours, the broker, finding the bill to be a counterfeit, returns and demands $50 of good money from the merchant. How much did the merchant lose by the operation?

REMARK.—At first glance some say $45 and the boots; some, $50 and the boots; some, $95 and the boots; and others, $100 and the boots. Which is correct?

14. What relation to me is my mother's brother-in-law's brother, provided he has but one brother?

15. Three men, travelling with their wives, came to a river which they wished to cross. There was but one boat, and but two could cross at one time; and, since the husbands were jealous, no woman could be with a man unless her own husband was present. In what manner did they get across the river?

REMARK.—Parke states that this problem "is found in the works of Alcuin, who flourished a thousand years ago."

16. Suppose it were possible for a man, in Cincinnati, to start on Sunday noon, when the sun is in the meridian, and travel westward with the sun, so that it might be in his meridian all the time. He would arrive at Cincinnati next day at noon. Now, it was Sunday noon when he started, it has been noon with him all the way around, and is Monday noon when he returns. The question is,

at what point did it change from Sunday noon to Monday noon?

17. Suppose a hare is 10 rods before a hound, and that the hound runs 10 rods while the hare runs 1 rod. Now when the hound has run the 10 rods, the hare has run 1 rod; hence they are now 1 rod apart, and when the hound has run that 1 rod, the hare has run $\frac{1}{10}$ of a rod; hence they are now $\frac{1}{10}$ of a rod apart, and when the hound has run the $\frac{1}{10}$ of a rod, they are $\frac{1}{100}$ of a rod apart; and in the same way it may be shown the hare is always $\frac{1}{10}$ of the previous distance ahead of the hound; hence the hound can never catch the hare. How is the contrary shown mathematically?

18. Think of any three numbers less than 10. Multiply the first by 2, and add 5 to the product. Multiply this sum by 5, and add the second number to the product. Multiply this last result by 10, and add the third number to the product; then subtract 250. Name the remainder, and I will name the numbers thought of, and in the order in which they were thought of.

METHOD.—The three digits composing this remainder, will be the numbers thought of; and the order in which they were thought of will be the order of hundreds, tens and units.

19. Write 24 with three equal figures, neither of them being 8.

METHOD.—$22 + 2 = 24$, or $3^3 - 3 = 24$.

20. Put down four marks, and then require a person to put down five more marks, and make ten.

METHOD.—The four marks are as represented in the margin; the five more, making ten, are placed as in the margin.

| | | ||
T E N

21. Which is the greater, and how much, six dozen dozen, or one-half a dozen dozen, or is there no difference between them?

22. Show what is wrong in the following reasoning:— $8 - 8$ equals $2 - 2$; dividing both these equals by 2

2 and the results must be equal; 8—8 divided by 2 —2=4, and 2—2 divided by 2—2=1; therefore, since the quotients of equals divided by equals, must be equal, 4 must be equal to 1.

23. A man has a triangular lot of land, the largest side being 136 rods, and each of the other sides 68 rods: required the value of the grass on it, at the rate of $10 an acre.

REMARK.—The "catch" in this is, that the sides given will form no triangle.

24. Says A to B, "Give me four weights, and I can weigh any number of pounds not exceeding 40." Required the weights and the method of weighing.

ANSWER.—The weights are 1, 3, 9, and 27 pounds. In weighing, we must put one or more in both scales, or some in one scale and some in the other; thus, 7 lbs. = 9 lbs. + 1 lb. — 3 lbs.

25. Mr. Frantz planted 13 trees in his garden, in such a manner that there were 12 rows, and only 3 trees in each row. In what manner were they planted?

ANSWER.—They were in the form of a regular hexagon, having a tree in the centre, and one at the middle and extremity of each side.

26. A and B raised 749 bushels of potatoes on shares; A was to have $\frac{3}{7}$, and B $\frac{4}{7}$ of them. Before they were divided, however, since A had used 49 bushels, B took 28 bushels from the heap, and then divided the remainder according to the above agreement. Was this division fair? if not, show how it should have been.

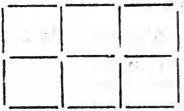
27. Two-thirds of six is nine, one-half of twelve is seven,
The half of five is four, and six is half of eleven.

SOLUTION.—Two-thirds of SIX is IX; the upper half of XII is VII; the half of FIVE is IV; and the upper half of XI is VI.

28. Does the top of a carriage-wheel move faster than the bottom? If so, explain it.

29. Supposing there are more persons in the world than any one has hairs on his head, there must be, at least, two persons who have the same number of hairs on the head, to a hair. Show how this is.

30. Place 17 little sticks—matches, for instance—making 6 equal squares, as in the margin. Then remove 5 sticks, and leave 3 perfect squares of the same size.



31. Three persons own 51 quarts of rice, and have only two measures; one a 4 quart, the other a 7 quart measure. How shall they divide it into three equal parts?

Method.—Perhaps the easiest way is to give each one 17 quarts, which may be obtained thus: fill the 7 quart measure; empty this into the 4 quart measure, and there will be 3 quarts in the 7 quart measure, which added to two 7 quart measures, equals 17 quarts.

32. What four United States coins will amount to fifty-one cents?

Answer.—Two 25 ct. pieces and two half-cents.

33. How may the nine digits be arranged in a rectangular form, so that the sum of any row, whether horizontal, vertical, or diagonal, shall equal 15?

4	9	2
3	5	7
8	1	6

Answer.—As in the margin.

34. How may the first 16 digits be arranged, so that the sum of the vertical, the horizontal, and the two oblique rows may equal thirty-four?

1	16	11	6
13	4	7	10
8	9	14	3
12	5	2	15

Answer.—As in the margin.

35. In what manner may the first 25 digits be arranged, so that the sum of each row of five figures may be 65?

1	10	12	18	24
9	11	20	22	3
13	19	21	5	7
17	23	4	6	15
25	2	8	14	16

ANSWER.—As in the margin.

REMARK.—The above are called Magic Squares. They are very interesting, and have engaged the attention of some of our greatest mathematicians, among whom we may mention Leibnitz, Stifels, &c. The methods of arrangement given above are by no means the only ones that may be used. For the second problem, Frenicle, a French mathematician, has shown that there may be 878 different arrangements.

36. Take 10 pieces of money, lay them in a row, and require some one to put them together in heaps 2 in each, by passing each piece over 2 others.

METHOD.—Let the pieces be represented by the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Place 7 on 10, 5 on 2, 3 on 8, 1 on 4, and 9 on 6.

37. An old Jew took a diamond cross to a jeweller, to have the diamonds reset; and fearing that the jeweller might be dishonest, he counted the diamonds, and found that they numbered 7 in three different ways. Now the jeweller stole two diamonds, but arranged the remainder so that they counted 7 each way, as before. How was it done?

METHOD.—The form of the cross when left is represented by Fig. 1, and when returned by Fig. 2. It will be seen by the figures how the diamonds were counted by the old Jew, and how they were arranged by the jeweller, who "jewed" the Jew.

Fig. 1.	Fig. 2.
7	7
6	767
76567	5
4	4
3	3
2	2
1	1

38. Let a person select a number greater than 1 and not exceeding 10; I will add to it a number not exceeding 10, alternately with himself; and, although he has

the advantage in selecting the number to start with, I will reach the even hundred first.

METHOD.—I make my additions so that the sums are, respectively, 12, 23, 34, 45, &c., to 89, when it is evident I can reach the hundred first. With one who does not mistrust the method, I need not run through the entire series, but merely aim for 89, or, when the secret of this is seen, for 78, then 67, &c.

39. Let a person think of any number on the dial-face of a watch; I will then point to various numbers, and at each he will silently add *one* to the number selected, until he arrives at *twenty*, which he will announce aloud, and my pointer will be upon the number he selected.

METHOD.—I point promiscuously about the face of the watch until the eighth point, which should be upon "12;" and then pass regularly around towards "1," pointing at "11," "10," "9," &c., until "twenty" is called, when, as may be easily shown, my pointer will be over the number selected.

40. Is there any difference between the results of the two following problems, and if so, what is it? If the half of 6 be 3, what will the fourth of 20 be? If 3 be the half of 6, what will be the fourth of 20?

41. A vessel with a crew of 30 men, half of whom were black, became short of provisions; and, fearing that unless half the crew were thrown overboard all would perish, the captain proposed to the sailors to stand upon deck in a row, and every ninth man be thrown overboard until half the crew were destroyed. It so happened that the whites were saved. Required the order of arrangement.

ANSWER.—W. W. W. W. B. B. B. B. B. W. W. B. W. W. W. B. W. B. B. W. W. B. B. B. W. B. B. W. W. B.

This can easily be found by trial, using letters or figures to represent the men.

42. Think of a number, multiply it by 6, divide this product by 2, multiply by 4, divide by 3, add 40, divide by 4, subtract the number thought of, divide by 2, and the quotient is 5. Show why this is so.

43. A and B were engaged by a Chester Co. farmer to dig 100 rods of ditch for $100; and since the part which A was to dig was more difficult of excavation than that which B dug, it was agreed that A should receive 10 shillings per rod, and B 6 shillings per rod. They each received $50 for their labor. How many rods did each dig?

REMARK.—This problem is usually regarded as incapable of solution, incorrectly so, however; for the results obtained by a mathematical process, rigidly accurate, are, A dug 37½, and B 62½ rods.

The majority of these puzzles and problems being founded upon principles quite easily comprehended, the author has not thought it necessary to explain the principles of the puzzles nor solve the problems. It is hoped that they may prove a source of pleasure and profit to teacher and pupil.

THE END.